UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER LABORATORY

NEW COMPUTER LIBRARY ROUTINE B3-EXP1-25

TITLE: exponential

TYPE: closed, relocatable, mnemonic

LENGTH: 30 words

TEMPORARY STORAGE: ten words at fixed memory locations 0-9 and the last two words of the program.

DURATION: approximately 800 microseconds (October 1962)

FAST REGISTERS CHANGED: none

PARAMETERS: link in M15

ACCURACY: relative error $< 2^{-40}$ for $-\ell n4 < z < \ell n4$

USE: This subroutine replaces the number z in the accumulator by

$$0 \quad \text{if} \quad z < -64\ \ell n4 < -87$$

$$e^z \quad \text{if} \quad -64\ \ell n4 < z < 64\ \ell n4$$

$$0 \text{ and sets OV} \quad \text{if} \quad z > 64\ \ell n4 > 87$$

DESCRIPTION: For $0 < x < \ell n4$ the formula $e^x = 1 + x/(1-x/2+F)$ is used where F is the continued fraction $F = \frac{a_1|}{|1} + \frac{a_2|}{|1} + \frac{a_3|}{|1} + \ldots,$ $a_n = x^2/(4(4n^2 - 1))$, which is computed to eight terms. For $z \geq \ell n4$, $e^z = 4^h e^x$, where h = integer part $(z/\ell n4)$, and $0 \leq x < \ell n4$. For negative values of z, e^z is computed as $1/e^{-z}$.

COMPARE: N. Macom, "On the computation of exponential and hyperbolic functions using continued fractions." JACM, Oct '55

DATE: November 14, 1962
PROGRAMMED BY: J. Nievergelt

EXP

0	SFR7,0	
	CAM13,0	
	CAM15,2,1	(M15) = address of Z
1	JDC2,0,2R	jump if Z ≥ 0
	CJF13,3	if Z negative, (M13) = 1
	STN15,0	and replace Z by |Z|
2	STR15,1	|Z| → memory 1
	DIV8,3,27R	Z/ln4
	SUB9,3,64	
3	JDC3,1,5R	jump if Z/ln4 < 64
	CAD15,3	Accu = e^Z = 0
4	JUM13,2,25R (exit)	jump if Z was negative
	DIV15,3	set OV if Z was positive
	JDC0,2,25R (exit)	
5	ADD9,3,64	reset Z/ln4
	SIA12,0	h = integer part (Z/ln4) → M12
6	AND8,3,26R	fractional part f in accu
	MPY8,3,27R	x = f ln 4
7	STR15,1	x → memory 2
	MPY10,3,2048	
	STR15,0	x/2 → memory 3
8	MPY15,1	
	STR8,3,28R	$x^2/4$ → memory 28R
	DIV9,3,3	
9	STR8,3,29R	a = $(x^2/4)/3$ → memory 29R
	CAD9,3,4	
10		initial value Δ'' = 4 → memory 4
	STR15,1	
	CAD9,3,3	initial value d = 3 → memory 5
11	STR15,1	
	CAD9,3,1	initial value
	STR15,1	$A_{-1} = B_0 = 1$
12	STR8,3,9	
	CAD15,3	initial value
	STR15,1	$A_0 = B_{-1} = 0$

13	STR15,0	
	CSM14,2,7	loop counter
	(l) ⟶ CAM15,2,6	(M15) = address of A_{n-2}
14	CAD8,3,29R	a
	MPY15,1	
15	ADD15,0	$a A_{n-2} + A_{n-1} = A_n$
	XCH15,1	
	STR8,3,6	old $A_{n-1} \rightarrow A_{n-2}$
16	CAD8,3,29R	a
	MPY15,1	
	ADD15,0	$a B_{n-2} + B_{n-1} = A_n$
17	XCH15,0	old $B_{n-1} \rightarrow B_{n-2}$
	STR8,3,8	
	CAD9,3,8	$\Delta'' = 8$
18	CAM15,2,4	(M15) = address of Δ'
	ADD15,0	$\Delta' \longleftarrow \Delta' + 8$
19	STR15,1	
	ADD15,0	$d \longleftarrow d + \Delta'$
	STR15,1	(M15) => A_{n-2} <
	VID8,3,28R	$(x^2/4)/d \rightarrow a$
20	STR8,3,29R	
	(l) ⟵ CJU14,3,13R	
21	CAD8,3,7	F = A/B
	DIV8,3,9	
22	ADD9,3,1	F + 1
	SUB8,3,3	F + 1 − (x/2)
23	VID8,3,2	
	ADD9,3,1	$e^x = 1 + x/(F + 1 - x/2)$
24	ADE12,0	$e^Z = 4^h e^x$
	JZM13,2,25R	jump if Z was positive
25	VID9,3,1	
	(exit) ⟶ LFR7,0	
	JLH15,0	
26	OCT0,17777,17777,17777	
27	OCT02613,11027,17372,03401	ln 4