Linear Programming—Feed Manufacturing

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Data Processing Application

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PREFACE

This manual supplies basic information on the use of linear programming to solve feed manufacturing problems. Its primary purpose is to introduce feed manufacturers to the advantages provided by linear programming applications already in use. However, the principles formulated here are geared to the interest of the entire agricultural community. Specifically, the manual provides an analysis of the feed manufacturing problem and illustrates the application of linear program procedures by constructing and solving a sample problem. The content is based on research performed by the Department of Agricultural Economics, School of Agriculture, Purdue University. For a discussion of the basic principles of linear programming and definitions of terms, refer to the IBM data processing application manual An Introduction to Linear Programming (E20-8171).

CONTENTS

INTRODUCTION

The use of linear programming (LP) in feed manufacturing necessitates a brief description of the nature of the industry, its key decisions, the use of the LP technique in making those decisions, and the potential advantages to be gained from such application. Particular emphasis is given to the use of the LP technique to:

- Reduce raw material procurement cost
- Minimize feed formula cost
- Evaluate sensitivity of feed formulas to ingredient price changes
- Formulate feed of uniform quality
- Evaluate the cost of formulation specifications
- Reduce inventory holding costs
- Determine most profitable product mix

Linear programming is not a stranger to modern management. Its use first became commercially significant in the early fifties, and the technique has become increasingly important in the last decade as a superior method of allocating the resources of business to achieve maximum profit or minimum cost. In fact, LP has become an indispensable aid to many firms that have successfully employed the technique in making important decisions.

Much of the effectiveness of the technique depends on how skillfully it is used. In this respect, the petroleum industry probably has led all others in the use of LP for gasoline blending and refinery scheduling. Additional areas of successful commercial application include the following: meat packing, milk processing, steel fabricating, airline scheduling, etc. (see reference 1).

Within the feed manufacturing industry, many of the major manufacturers use LP to reduce formula costs. In fact, companies that do not use the technique frequently cannot be cost-competitive with those that do. Moreover, LP users generally have better product quality control because of the technical knowledge they have gained from programmed analysis of feed formulas.

Despite these advantages, the use of LP in the feed manufacturing industry has been relatively limited and restricted mainly to formulation problems In fact, in many respects its application is still in its infancy. Fully effective use has been hindered by several major factors.

First, a lack of understanding has prevailed as to the amount of mathematical proficiency needed to use the technique. Despite popular impressions, it is not necessary to have a staff of highly trained LP specialists. It is important to have people who can understand the basic elements of what the technique does, how problems can be formulated, and what all linear programming solutions mean in terms of management decisions. But all this does not require a high degree of mathematical dexterity. Instead, the main requirement for successful LP use is an intimate knowledge of the problem to be solved. Accurate representation must be given to the physical operations considered, to the economic performance criteria used, and to the overall managerial policies of the firm. Within such context, this manual is designed to provide the necessary background for development of LP models.

Second, the scarcity of good computer programs (LP systems) that are needed for problem analysis and solution has restricted LP use. As a consequence, it has been necessary for individual firms to build their own codes or to use inadequate library programs. Both alternatives are costly and inhibiting. However, good LP codes now are available as application programming packages for IBM computing systems. In all cases, LP systems provide management with the necessary programming support for immediate use of the technique.

PROBLEM PROFILE

The feed formulation problem is one of determining the least-cost combination of feedstuff ingredients that can be used to meet predetermined product formulation specifications. The dimensions of the problem vary widely from company to company depending upon such factors as size of firm, ingredient availability, operating capacity, product mix, and market position. For many firms, the problem is closely interrelated with raw material procurement policy, inventory policy, production scheduling, and product mix and price strategies. In such cases, the basic LP formulation model can be expanded to include this wide range of interdependent decision areas.

The major part of this manual deals with the basic formulation problem. The interrelated areas also are discussed but in less detail.

PROBLEM ECONOMICS

The use of LP makes savings possible in the numerous areas mentioned above. The most apparent benefit is the direct reduction in feed formula cost, which could run at a minimum of $2 per ton (see reference 3). This saving does not include the economic benefits accruing from improved feed quality and uniformity, or the longer-run benefits accruing from more accurate experimental formulation data.

Increased efficiency and allied benefits in other areas of feed manufacturing related to formulation also are important. Those most obvious include cost reductions in raw material procurement, inventory expense, and production scheduling. Normally, an LP formulation model of the major manufacturing operations includes over 90 percent of the costs associated with those operations, and the model can bring about cost reduction of up to $4 per ton of feed manufactured.

PROBLEM DESCRIPTION

The first problem analyzed is that of determining the least-cost formula for a complete broiler ration. Of the 18 ingredients or feedstuffs that can be used in this ration, nine must fall within specified minimum and/or maximum rates of utilization. The remaining feedstuffs can be used in any proportion desired as long as the other formulation specifications are met. Those specifications consist of 18 requirements relating to such factors as energy level, protein, fat, fiber, and amino acids.

The ingredients are assumed to be available in unlimited quantities at prevailing market prices. Delivery charges, processing, handling, and other costs associated with using ingredients directly from purchase or out of inventory are included in the ingredient costs used in the model.

The second problem analyzed is that of determining least-cost formulas for a series of cattle, dairy, and hog supplements. In this case the model is similar to that developed initially but is designed to analyze sequentially a series of distinct formulas using one basic matrix. The model permits a greatly reduced computation time per formula and is the type that would be used on an operating basis by feed manufacturers.

The third problem involves the development of a multiformula model where availability of ingredients is limited. The same basic model can be used to analyze manufacturing operations with other restrictions, such as limitations in production and storage capacity.

The fourth problem also involves a multiformula model with various production and procurement restrictions. In addition, the problem is designed to analyze decisions relating to product mix and pricing strategy.

TYPES OF LP MODELS

A linear programming model is developed for each of the four types of managerial problems outlined in the preceding section. The models are mathematical representations of all known and estimated factors that influence the cost and/or profit of selected operations in feed manufacturing. They are representative of types of LP models currently being used within the industry, and they have proven commercial feasibility.

The following discussion considers first the types and sources of data that are needed to build LP models. Then, it concerns itself with the attendant analytical procedures of model building and the development of explicit management guides based on the use of those models.

SINGLE FORMULA MODEL

The single formula model is designed to determine the least-cost combination of ingredients that can be used to meet predetermined product specifications. Today the most widespread application of LP in the feed manufacturing industry involves variations of this model. Because it is initially simple in concept, the single formula model forms the basis for development of more elaborate models. In this sense it is a necessary starting point for programmed analysis of feed formulation and related areas of manufacturing and marketing.

INFORMATION NEEDED

At minimum, three major areas must be considered in the initial analysis: (1) ingredient availability and cost, (2) ingredient composition, and (3) formulation specifications for feed products. Much of the basic data is available from purchasing, accounting, and nutritional research. But problems exist involving variability in ingredient composition, rapidly changing ingredient prices, and competitive specifications for feed formulas. Moreover, it is important that such data accurately reflect company practice and policy relating to procurement, manufacturing, and marketing. Significant discrepancies between the mathematical representation of the problem and actual company policy can destroy the effectiveness of the system.

INGREDIENT COST AND AVAILABILITY

For purposes of discussion, 18 ingredients are assumed to be available for feed formulation (Figure 1). Supplies are adequate to meet the production needs of any ingredient. Two price situations are analyzed; they are referred to hereafter as "week one" and "week two" prices (Figure 1).

Least-cost formula guides are computed for each price situation, and the computations, in turn, demonstrate the sensitivity of feed cost and formula changes to changes in ingredient prices. Ingredient prices are based on management's best estimate of net procurement cost or disposal value, adjusted for delivery charges, handling, and other associated costs.

INGREDIENT COMPOSITION

Variation in composition is a continuing problem in feed formulation. In particular, variations in the levels of protein, fiber, amino acids, and certain physical characteristics are of prime importance. However, such problems are not unique to LP formulation. They must be faced without regard to the methods used in determining formulas. Composition measurement and control of ingredient quality are a difficult but necessary part of effective LP use.

Ingredient	Identification	Week One ($/ton)	Week Two ($/ton)
Alfalfa Meal	X1	61.00	63.00
Barley Grain	X2	63.00	64.00
Corn Meal	X3	54.00	52.00
Corn Distillers Solubles	X4	75.00	72.00
Corn Gluten Meal	X5	86.00	84.00
Crab Meal	X6	54.00	50.00
Dicalcium Phosphate	X7	72.00	74.00
Fishmeal (Menhaden)	X8	125.00	127.00
Limestone	X9	9.00	10.00
Meatscraps	X10	88.00	87.00
Milo	X11	46.00	48.00
Methionine	X12	2100.00	2400.00
Oats	X13	52.00	50.00
Poultry Feed	X14	103.00	94.00
Soybean Meal	X15	84.00	75.00
Stabilized Fat	X16	138.00	142.00
Whey	X17	125.00	115.00
Salt, Vitamins & Minerals	X18	575.00	575.00

Figure 1. Ingredient prices

In this analysis, it is assumed that ingredient composition is constant (Figure 2), but in actual practice, management will have to update composition indices periodically. Modern instrumentation and sampling techniques permit accurate and economic measurement of the nutrient content of ingredients. However, the cost of determining nutrient content must be balanced against the loss experienced when a safety margin is added to minimum nutritional requirements, and this will vary widely with individual ingredients and requirements.

FORMULATION SPECIFICATIONS

Despite fundamental agreement within the industry on necessary formulation specifications, obvious differences of opinion exist among competing companies and nutritional experts. Generally recommended nutrient levels may not meet the needs of a particular market area or of competitive strategy. Non-nutrient standards also may vary from firm to firm.

In evaluating the given area, the LP analyst must gain consensus within his own company. All standards must be clearly specified. The specifications that are built into the LP model must not err on either side of desired or actual operating practice. In later sections, the role of LP in determining optimum formulation specifications is discussed.

	METEN[a]	PRDEN[a]	FAT[b]	FIBER[b]	PROTN[b]	CALCM[b]	PHOSP[b]	ARGIN[b]	GLYCN[b]	LYSIN[b]	METHN[b]	MECYS[b]	TRYPT[b]	XANTH[c]
ALFML	530.	350.	2.5	18.0	20.0	1.7	.09	.9	.9	.9	.9	.3	.34	110.
BARLY	1280.	800.	1.8	8.0	11.5	.6	.13	.5	.35	.35	1.7	.34	.13	--
CORN	1580.	1145.	3.8	2.5	8.6	.06	.24	.4	.29	.20	.16	.32	.08	7.
DISTS	1320.	1020.	6.	4.	27.	.35	.46	.9	.9	.9	.6	1.	.2	--
GLTML	1170.	840.	1.5	4.5	42.0	.1	.13	1.3	1.4	.75	.93	1.6	.2	60.
CRBML	850.	600.	2.2	10.5	31.1	14.6	1.5	1.45	1.8	1.2	.44	.54	.25	--
DPHOS						26.	18.							
FSHML	1350.	920.	7.5	--	60.0	5.0	2.9	3.9	3.8	5.1	1.8	2.8	.61	--
LIMST						38.5								
MEATS	920.	760.	6.	2.5	55.0	8.0	4.0	3.8	7.4	3.3	.64	1.25	.35	--
MILO	1410.	1110.	2.5	2.5	9.5	.04	.1	.34	.32	.28	.16	.34	.15	--
METHN											100.	100.		
OATS	1140.	760.	4.	12.	12.	.1	.11	.7	.38	.31	.16	.31	.15	--
PLTML	1280.	900.	12.	2.5	55.	3.	1.7	3.4	2.6	3.4	.96	1.91	.45	
SOYML	1120.	650.	.5	3.	51.	.25	.2	3.6	2.7	3.2	.75	1.5	.6	
STFAT	3500.	2900.	100.											
WHEY	830.	700.	.5	--	12.	.8	.22	.36	.7	.35	.3	.62	.18	

a is in calories per pound; b is in percent; c is in milligrams per pound.

Legend			
METEN	Metabolizable energy	ALFML	Alfalfa Meal
PRDEN	Productive energy	BARLY	Barley
FAT	Fat	CORN	Corn
FIBER	Fiber	DISTS	Corn Distillers Solubles
PROTN	Protein	GLTML	Corn Gluten Meal
CALCM	Calcium	CRBML	Crab Meal
PHOSP	Phosphorus	DPHOS	Dicalcium Phosphate
ARGIN	Arginine	FSHML	Fish Meal Menhaden
GLYCN	Glycine	LIMST	Limestone
LYSIN	Lysine	MEATS	Meatscraps
METHN	Methionine	MILO	Milo
MECYS	Methionine and Cystine	METHN	Methionine
TRYPT	Tryptophane	OATS	Oats
XANTH	Xanthophyll	PLTML	Poultry Meal
		SOYML	Soybean Meal
		STFAT	Stabilized Fat
		WHEY	Whey

Figure 2. Ingredient composition, single formula model

Nutrient standards of the formula are defined in terms of such factors as protein (crude), fat, fiber, amino acids, and energy. Most of these requirements are met by an acceptable combination of ingredients. However, vitamin and mineral requirements usually are met only partially by feed ingredients. For purposes of simplification, it is assumed in this example, that a fixed amount of salt, vitamin, and mineral premixes is to be added to the ration to guarantee that minimum needs are met in those areas.

Where nutrient standards are not well defined, manufacturers in certain cases have specified, as a precautionary measure, minimum levels of use of key ingredients. For example, several unidentified growth factors can be expressed in terms of minimum levels of fish meal, whey, alfalfa meal, or other ingredients.

Non-nutrient specifications also are important, since they control the appearance, odor, palatability, and texture of the product. Ideally, it would be desirable to have accurate indices of the levels of the non-nutrient factors in each ingredient. Unfortunately, however, to date such indices have not been precisely quantified. As a workable approach in the interim, non-nutrient standards are met through the imposition of maximum and/or minimum use levels of specific ingredients.

MODEL FORMULATION

The difficult task of model building lies in obtaining an accurate definition of the actual operating conditions under study. Careful calculation of the relevant cost, composition, and formulation specifications data usually is more time consuming than the mathematical statement of the model or the actual processing of the model data.

For the problem discussed above, the LP model or matrix is stated as shown in Figure 3. Each of the ingredients used in feed production is expressed as a matrix column or activity; formulation specifications are stipulated by the rows and elements of the right-hand-side (RHS) column.

For initial analysis, production of 100 tons of feed is assumed. The cost row coefficients are expressed in terms of dollars per ton. With the exception of energy and xanthophyll requirements, coefficients of the row equations are expressed in terms of percent or portion of process volume of feed produced. For energy control, row coefficients are scaled to give control in terms of calories per pound for each pound of feed in the total volume processed. For xanthophyll control, row coefficients are scaled to give control in terms of milligrams per pound for each pound of feed in the total volume processed.

As formulated, the model provides for finding the minimum of the cost row function (formula cost), subject to the formulation specification constraints. The formula cost and formulation specifications are expressed as functions of the amounts of ingredients used. Ingredient utilization is represented by the symbols X_1.

Formula Cost Control

The cost per ton for each ingredient is given in the cost row. This row constrains the formula chosen to that combination of ingredients that satisfies all formulation specifications at the minimum possible cost. The equation may be stated as follows:

$$61.00\ X_1 + 63.00\ X_2 + \ldots\ldots + 575.00\ X_{18} = \text{minimum} \tag{1-1}$$

Thus, alfalfa meal cost ($61) times amount used ($X_1$) plus barley cost ($63) times amount used (X_2) plus ..... plus salt, vitamin, and mineral premix cost ($575) times amount used ($X_{18}$) must equal a minimum.

	Alfalfa Meal (20%)	Barley	Corn Meal	Corn Distillers Solubles	Corn Gluten Meal	Crabmeal	Dicalcium Phosphate	Fishmeal Menhaden	Limestone	Meatscraps	Milo	Methionine	Oats	Poultry Meal	Soybean Meal	Stabilized Fat	Whey	Salt, Vit, Min		
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}		
Week One Cost	61.	63.	54.	75.	86.	54.	72.	125.	9.	88.	46.	2100.	52.	103.	84.	138.	125.	575.		
Week Two Cost	63.	61.	52.	72.	84.	50.	74.	127.	10.	87.	48.	2400.	50.	94.	75.	142.	115.	575.		
Weight Control	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	=	100.
Metabolizable Energy Mn	6.4	12.8	15.8	13.2	11.7	9.0		13.5		9.2	14.1		11.4	12.8	11.2	35.	8.3		≥	1400.
Productive Energy Mn	3.9	8.	11.45	10.2	8.4	7.5		9.2		7.6	11.1		7.6	9.	6.5	29.	7.		≥	1000.
Total Fat Mn	.025	.018	.038	.06	.015	.022		.075		.06	.025		.04	.12	.005	1.	.005		≥	6.
Added Fat Mx																1.			≤	8.
Fiber Mx	.18	.08	.025	.04	.045	.105		1.		.025	.025		.12	.025	.03				≤	5.
Protein Mn	.20	.115	.086	.27	.42	.311		.6		.55	.095		.12	.55	.51		.12		≥	22.
Calcium Mn	.017	.006	.0006	.0035	.001	.146	.26	.05	.385	.08	.00036		.001	.03	.0025		.008		≥	1.
Calcium Mx	.017	.006	.0006	.0035	.001	.146	.26	.05	.385	.08	.00036		.001	.03	.0025		.008		≤	1.1
Phosphorus Mn	.0009	.0013	.0024	.0046	.0013	.015	.18	.029		.04	.001		.0011	.017	.002		.0022		≥	.45
Phosphorus Mx	.0009	.0013	.0024	.0046	.0013	.015	.18	.029		.04	.001		.0011	.017	.002		.0022		≤	.6
Arginine Mn	.009	.005	.004	.009	.013	.0145		.039		.038	.0034		.007	.034	.036		.0036		≥	1.2
Glycine Mn	.009	.0035	.0029	.009	.014	.018		.038		.074	.0032		.0038	.026	.027		.007		≥	.84
Lysine Mn	.009	.0035	.002	.009	.0075	.012		.051		.033	.0028		.0031	.034	.032		.0035		≥	1.1
Methionine Mn	.003	.0017	.0016	.006	.0093	.0044		.018		.0064	.0016	1.	.0016	.0096	.0075		.003		≥	.5
Methionine & Cystine Mn	.0065	.0034	.0032	.01	.016	.0054		.028		.0125	.0034	1.	.0031	.0091	.015		.0062		≥	.85
Tryptophane Mn	.0034	.0013	.0008	.002	.002	.0025		.0061		.0035	.0015		.0015	.0045	.006		.0018		≥	.22
Xanthophyll Mn	1.2		.07	.008	.6														≥	6.3
Barley, Milo Mx		1.									1.								≤	15.
Corn Distillers Sol Mx				1.															≤	5.
Corn Gluten Mx					1.														≤	5.
Crab Meal Mx						1.													≤	5.
Fish Meal Mx								1.											≤	7.5
Poultry Meal Mx														1.					≤	7.5
Salt, Vit, Min																		1.	=	.68

Figure 3. Matrix tableau, single formula model

Process Volume

Process volume may be set at any desired level. Frequently, computations are made on a per ton or per pound basis. In this analysis, a 100-ton volume is used. Computer solutions may be interpreted in terms of tons of different ingredients in 100 tons of feed, or in terms of percentages of different ingredients in 100 percent of a ton of feed. This volume control may be formulated as follows:

$$X_1 + X_2 + \ldots\ldots + X_{18} = 100 \qquad (1\text{-}2)$$

That is, amount of alfalfa meal used (X_1) plus amount of barley used (X_2) plus ..... plus amount of salt, vitamin, and mineral premix used (X_{18}) shall equal 100 tons.

Fat Content

Fat content is specified at a minimum level of six percent of finished product. Thus, for every 100 tons of feed produced, there must be a minimum fat level of six tons. The constraint is stated as follows:

$$.025\ X_1 + .018\ X_2 + \ldots\ldots + .005\ X_{17} \geq 6 \qquad (1\text{-}3)$$

This specifies that fat level (2.5 percent) in alfalfa meal times amount used (X_1) plus fat level (1.8 percent) in barley meal times amount used (X_2) plus ..... plus fat level (.5 percent) in whey times amount used (X_{17}) shall be equal to or greater than six tons out of the total production volume of 100 tons. For purposes of computer analysis, it may be necessary to convert the inequality statement (1-3) to an equality statement as follows:

$$.025\ X_1 + .018\ X_2 + \ldots\ldots + .005\ X_{17} - XNS = 6 \qquad (1\text{-}4)$$

This specifies that the amount of fat in the ingredients used ($X_1 \ldots\ldots X_{17}$) less any excess (X negative slack or XNS) is equal to six. Any acceptable combination of ingredients must contain at least six tons of fat. If a least-cost combination of ingredients contains more than six tons, the negative slack variable (XNS) takes the value of the excess, thereby maintaining a balance in the fat control constraint. Other equal to or greater than inequalities are handled in a similar manner.

Generally, IBM LP systems accept inequalities and automatically generate slacks.

Protein Control

Crude protein content is specified at a minimum level of 22 percent of finished product or 22 tons of crude protein for every 100 tons of broiler ration produced. This control is established as follows:

$$.20\ X_1 + .115\ X_2 + \ldots\ldots + .12\ X_{17} \geq 22 \qquad (1\text{-}5)$$

In other words, protein level (20 percent) in alfalfa meal times amount used (X_1) plus protein level (11.5 percent) in barley meal times amount used (X_2) plus ..... plus protein level (12 percent) in whey times amount used (X_{17}) shall be equal to or greater than 22 tons for every 100 tons of feed produced.

Other Nutrient Controls

Other nutrient standards are handled in a similar manner. Note that most requirements are stated in terms of minimum levels of the various nutrient standards. However, maximum constraints are necessary for such items as fiber and calcium.

Individual Ingredient Controls

Utilization rates of individual ingredients may be controlled for nutritional or non-nutritional reasons. Generally, such controls are concerned with appearance, flavor, texture, odor, and possibly the growth factor. This type of constraint may be expressed as follows:

Barley, milo control

$$X_2 + X_{11} \leq 15 \qquad (1\text{-}6)$$

That is, barley used (X_2) plus milo used (X_{11}) shall be equal to or less than 15 tons. This inequality statement (1-6) is converted to an equality statement as follows:

$$X_2 + X_{11} + XPS = 15 \qquad (1\text{-}7)$$

This specifies that the amount of barley (X_2) plus the amount of milo (X_{11}) plus the positive slack (XPS) must equal 15. Thus, barley and/or milo may constitute up to 15 percent of the ration. If the ingredients account for less than 15 percent, the positive slack variable takes the value of the difference, thereby maintaining the balance in the barley, milo control equation.

Premix Control

Salt, vitamin, and mineral premix utilization is set at .68 tons per 100 tons of feed manufactured. This specification ensures that all necessary salt, vitamin, and mineral needs are met. The control is expressed as follows:

$$X_{18} = .68 \tag{1-8}$$

PROBLEM SOLUTION

The solution to the LP problem described above yields a wide range of data useful in feed manufacturing, which in this initial model include the following:

1. Least-cost feed formula guides
2. Measurement of the sensitivity of least-cost formulas to changes in ingredient prices
3. Ingredient procurement guides
4. Measurement of the cost of formulation specifications

Too frequently, LP users are concerned solely with the basic least-cost formula data, while much or all of the collateral data is overlooked.

FORMULA GUIDES

Least-cost formula guides are supplied for each set of market conditions and formulation specifications analyzed. From these guides, management can quickly determine the particular formula that minimizes ingredient cost while meeting formulation specifications. For the problem under consideration, column 1 in Figure 4 lists the ingredients to be included in the least-cost formula; column 2 lists the cost of each ingredient to be used; and column 3 lists the percent of each ingredient to be used.

It may be seen that twelve of a possible 18 ingredients are included in the least-cost formula. Utilization rates range from a high of 43.51 percent for corn meal (X_3) to 1.17 percent (X_9) for limestone. Utilization rates of micro ingredients range from .68 percent for the premixes (X_{18}) to .15 percent for methionine (X_{12}).

Note that Figures 4 and 5 are reports generated entirely by the computer and that copies can be made available for management within minutes after the least-cost formula problem has been entered into the machine. These particular reports stem from the IBM 1620 (4). Most other IBM machines have similar capacity.

In terms of the technical LP language, the amounts of ingredients specified for use are the activity levels or the values of the basis variables when an optimum solution has been obtained. These values may be read directly from the unedited computer output or may be transposed by clerical assistants. However, in most cases computer editing or report writing is the most effective way of getting decision guides that management can use directly.

LEAST-COST BROILER RATION
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	61.00	1.39	58.68	61.53
CORN MEAL	54.00	43.51	51.96	54.16
CORN GLUTEN MEAL	86.00	2.63	85.78	87.09
LIMESTONE	9.00	1.17	2.89	12.13
MEATSCRAPS	88.00	3.99	79.80	88.22
MILO	46.00	15.00	***	52.08
METHIONINE	2100.00	.15	1843.75	2110.78
OATS	52.00	4.68	51.88	53.51
POULTRY MEAL	103.00	7.50	***	106.25
SOYBEAN MEAL	84.00	16.79	82.55	84.14
STABILIZED FAT	138.00	2.49	106.00	141.05
SALT VITAMIN MINERAL	575.00	.68	***	575.00

FORMULA COST PER TON 71.87

Figure 4. Least-cost broiler ration

RESERVE INGREDIENT BUY GUIDE
DECEMBER 10,196-

FEED INGREDIENTS NOT IN FORMULA	PRICE PER TON	PENALTY COST	HIGHEST FEASIBLE PRICE
BARLEY	63.00	18.05	44.95
CORN DISTILLERS SOL	75.00	1.66	73.34
CRAB MEAL	54.00	.98	53.02
DICALCIUM PHOSPHATE	72.00	11.00	61.00
FISH MEAL MENHADEN	125.00	.15	124.85
WHEY	125.00	71.33	53.67

Figure 5. Reserve ingredient buy guide

Price Sensitivity

Sensitivity of least-cost formulas to changes in ingredient prices is indicated in Figure 4. These values are computed through a parametric analysis of ingredient costs and measure the amount of ingredient price change that can occur (one at a time) without necessitating a change in the formula.

Cost ranges are given for each feedstuff included in the formula (columns 4 and 5). For example, alfalfa meal, priced at $61 per ton, is used at the rate of 1.39 percent in the formula. This utilization rate remains least-cost as long as the alfalfa meal price does not go below $58.68 per ton or above $61.53 per ton. Prices below $58.68 per ton bring an increase in use of alfalfa meal. Prices above $61.53 per ton cause alfalfa meal to be dropped from the formula. Similar ranges are given for all other feedstuffs included in the least-cost formula.

Note that some ranges are much narrower than others, thereby indicating a higher degree of price sensitivity. Corn gluten meal, for example, must fall within a $1.31 price range if the current utilization rate of 2.63 percent is to remain optimum.

Other ranges may be relatively wide, but if either side of a given price range falls close to the market price, the particular ingredient becomes price sensitive. Meatscraps, for example, has a price range of $79.80 to $88.22 per ton for the 3.99 percent utilization rate. However, the market price is $88 per ton. Thus, a slight upward pressure on the market price causes meatscraps to be dropped from the ration.

In certain cases, either side of the range may be unlimited. For example, consider poultry meal, priced at $103 per ton, with a utilization rate of 7.5 percent. Recall that poultry meal use (X_{14}) is limited to a maximum level of 7.50 percent, that is:

$$X_{14} \leq 7.50 \quad (1\text{-}9)$$

Thus, the utilization rate in the least-cost ration already is at the maximum allowable level. The lower price range is unlimited (indicated by ***, Figure 4). This means that the 7.50 percent utilization rate cannot be increased regardless of how much the price is lowered. The upper range indicates that the 7.50 percent utilization rate continues only as long as the price does not exceed $106.25 per ton.

It is important to remember that price ranges are valid only if they are considered one at a time. Usually, this is not unduly restrictive, since major price fluctuations frequently center around only one or two key ingredients. This is particularly true if the major concern is with those ingredients that make up the bulk of the ration. Of course, when price changes are more widespread, it is necessary to compute a new least-cost formula.

Procurement Guides

Ingredient procurement guides are supplied for each feed formula specified. An aggregation of each formula times volume produced gives the overall total use required. This aggregate use can be tallied from individual formula models, or from a multiformula model (as explained in later sections). The related questions of safety stocks, inventory capacity, material-in-process, and delivery lag times, also can be included in LP models.

A second type of procurement information available from the single formula model is shown in Figure 5. Ingredients not included in the least-cost ration (nonbasis variables) are given along with the penalty cost (D/J values) of introducing those excluded ingredients. For example, the use of barley increases the formula cost by $.1805 per ton for each one percent used in the formula.

Obviously, such information is of value to management. Surpluses or shortages of available ingredients may raise the question of ingredient substitution. For example, fish meal does not come into the ration; however, if the ingredient is introduced to the formula, the increased cost is only $.15 per ton used. Thus, if other compelling reasons actually force its use, the penalty cost is sufficiently low to make it a desirable ingredient to include in the formula.

Furthermore, the penalty cost information also can be used for bargaining purposes. For example, given a market price of $75 per ton for corn distillers solubles, and a penalty cost of $1.66 per ton, the highest feasible price (basis value) to pay for corn distillers solubles is $73.34 per ton ($75.00 — $1.66). At $73.34 per ton or less it is a good buy. At any higher price it is cheaper to use some other ingredient in the ration.

Cost of Formulation Specifications

The costs of individual formulation specifications are indicated in Figure 6. These are of direct use both to the nutritionist and to sales personnel. Specifically, the information provides an accurate guide to the cost of meeting both nutrient and nonnutrient standards. Furthermore, the range over which the calculated cost of the constraint is applicable also is given.

Upper Limit Constraints

Upper limit constraints specify the maximum allowable level of use of an ingredient or of fiber or some other factor in the ration. For example, maximum use of poultry meal is limited to 7.5 percent; fiber content is held to a maximum level of five percent; etc.

Checking the computer solution values against the poultry meal restriction, we have the following:

Restriction

X_{14}	+ XPS	= 7.5	(1-10)
amount poultry meal used	positive slack	maximum amount poultry meal	

Computer Solution

$$(7.5) + (0) = 7.5 \quad (1\text{-}11)$$

Thus, the least-cost formula contains the maximum allowable amount of poultry meal.

Constraint	Amount	Slack	Unit Cost*	Minimum Value	Maximum Value
Mn Met Energy	14.00cal/lb	0	.49	1366.	1417.
Mn Prd Energy	1000.cal/lb	29.	0	***	1030.
Mn Fat	6.00%	0	87.29	5.256	10.430
Mx Fiber	5.00%	1.80	0	3.186	***
Mn Protein	22.00%	0	22.42	21.337	22.155
Mn Calcium	1.00%	.10	0	***	1.100
Mx Calcium	1.10%	0	63.32	1.000	1.550
Mn Phosphorus	.45%	0	244.93	.404	.494
Mx Phosphorus	.60%	.15	0	.450	***
Mn Arginine	1.20%	.12	0	***	1.316
Mn Glycine	.84%	.34	0	***	1.185
Mn Lysine	1.10%	0	58.48	1.084	1.149
Mn Methionine	.50%	.0033	0	***	.502
Mn Meth, & Cystine	.85%	0	2066.62	.848	2.019
Mn Tryptophane	.22%	.003	0	***	.223
Mn Xanthophyll	6.3mg/lb	0	3.90	4.476	8.358
Mx Barley, Milo	15.00%	0	6.08	6.555	23.788
Mx Corn Dist Sol	5.00%	5.00	0	0	***
Mx Corn Gluten	5.00%	2.37	0	2.634	***
Mx Crab Meal	5.00%	5.00	0	0	***
Mx Fish Meal	7.50%	7.50	0	0	***
Mx Poultry Meal	7.50%	0	3.25	0	8.551
Mx Stab Fat	8.00%	5.00	0	2.486	***
Salt, Premixes	.60%	0	541.62	0	1.849

* Cents per ton per unit of restriction.

Figure 6. Specification costs week one

The collateral information given by the computer may be summarized as follows:

Constraint	Required (RHS)	Slack	Constraint Cost (PI)	Range of Constraint Cost (Min)	(Max)
Mx Poultry Meal	7.50	0	3.25	0	8.551

Columns 1 and 2 indicate each constraint and its required amount.

Column 3 indicates whether the constraint has become restrictive. If the slack variable comes into solution at a zero value as in the case of poultry meal use, the constraint is limiting. If the slack variable comes into solution at a positive value, the constraint is not limiting.

Column 4 indicates the cost of including the restriction in the model and is known technically as the marginal cost. In the case of the poultry meal maximum constraint, the cost is 3.25 cents per percent per ton. That is, for each percent of relaxation in the constraint, the cost is decreased by 3.25 cents per ton.

Columns 5 and 6 indicate the range over which the cost is applicable. In this case, the 3.25 cent cost holds from zero utilization (Minimum) to utilization at the 8.551 percent level (Maximum). Similar information is given for the other upper limit constraints (Figure 6). It should be noted that many of the upper limits on ingredient utilization are not limiting.

Lower Limit Constraints

Lower limit constraints specify the minimum allowable level of use of an ingredient, or of fat, protein, or some other nutrient in the ration. Most of the restrictions are of this type and concern levels of fat, protein, energy, and amino acids. Consider the following:

Constraint	Amount Required	Slack	Constraint Cost	Range of Constraint Cost (Min)	(Max)
Fat	6.000	0	87.29	5.256	10.430
Protein	22.000	0	22.42	21.337	22.155
Glycine	.840	.340	0.00	***	1.185

Meeting the minimum fat requirement costs 87.29 cents per percent of the fat requirement per ton from a minimum of 5.256 percent to a maximum of 10.430 percent. Thus, lowering the fat requirement from six percent to five percent decreases per ton costs by 87.29 cents. Similarly, the protein constraint costs 22.42 cents per percent per ton from the 21.337 percent level to the 22.155 percent level. Other constraints, such as glycine (excess of .340), do not become limiting. In this case, the constraint is without cost from a range of zero up to the 1.185 percent level.

Price Variations

For purposes of illustration, the initial broiler ration problem was analyzed under conditions of week two prices. All other parameters are identical to those of the initial problem. Management guides, developed for the new price conditions, form a basis for measuring the impact of price changes.

Relative price changes between week one and week two are not very great. The overall price level is lower for week two, and the relative position of selected ingredients is changed. However, the overall pattern is typical of the types of changes that evolve from week to week.

MANAGEMENT GUIDES

Formula cost decreased from $71.87 per ton in week one (Figure 4) to $69.26 per ton in week two (Figure 7). Major ingredient changes included significant increases in soybean meal, alfalfa meal, and crab meal. Corn gluten meal was dropped from the ration, and utilization of oats was cut back sharply. Other changes were of more modest proportions.

LEAST-COST BROILER RATION
DECEMBER 17, 196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	63.00	2.70	53.09	65.72
CORN MEAL	52.00	43.73	50.44	54.42
CRAB MEAL	50.00	1.63	47.79	56.78
DICALCIUM PHOSPHATE	74.00	.72	34.34	103.92
LIMESTONE	10.00	.82	***	16.76
MILO	48.00	15.00	***	49.69
METHIONINE	2400.00	.15	1869.26	2854.22
OATS	50.00	1.93	48.17	51.49
POULTRY MEAL	94.00	7.50	***	98.38
SOYBEAN MEAL	75.00	22.37	71.87	76.83
STABILIZED FAT	142.00	2.77	110.24	203.04
SALT VITAMIN MINERAL	575.00	.68	***	575.00
FORMULA COST PER TON	69.26			

Figure 7. Least-cost broiler ration

The least-cost formula for week one is generally more sensitive to upward price pressure than is the formula for week two. The absolute price changes are of similar magnitudes for both weeks; however, week one upper limits are closer to market prices than are those of week two. Penalty costs (D/J values) for ingredient substitution are generally prohibitive in both weeks (see Figure 8).

RESERVE INGREDIENT BUY GUIDE
DECEMBER 17, 196-

FEED INGREDIENTS NOT IN FORMULA	PRICE PER TON	PENALTY COST	HIGHEST FEASIBLE PRICE
BARLEY	64.06	17.10	46.90
CORN DISTILLERS SOL	72.00	2.28	69.72
CORN GLUTEN MEAL	84.00	1.28	82.72
LIMESTONE	10.00	9.09	.91
MEATSCRAPS	87.00	5.22	81.78
WHEY	125.00	63.09	61.91

Figure 8. Reserve ingredient buy guide

Changes in the costs of nutrient formulation restrictions partially reflect decreases in the overall price level. However, increases in marginal costs occurred in the cases of fat, calcium, methionine, and xanthophyll. Changes in costs of nonnutrient specifications are traceable more directly to ingredient price changes (see Figure 9).

As a matter of practical application, if meaningful interpretation is to be given, it is necessary to analyze marginal costs over a series of weeks. For most feed formulas, some marginal costs are relatively stable, while for others they are very sensitive and unstable. Hence, the need for a composite analysis of values over a given period.

MODEL VARIATIONS

In addition to the usual model constraints discussed in connection with the single formula model, it may be necessary to use constraints of the following type: (1) ratio constraints, (2) dual constraints, (3) feed density constraints, and (4) blocking constraints. In general, these serve to increase the effectiveness of the LP technique.

Ratio Constraints

Ratio constraints are used to control the relative value of any two or more activities in the formulation model. For example, consider the control of the ratio between the nonplant origin, high protein ingredients and the plant origin, high protein ingredients. Assume the nonplant origin, high protein ingredients

Constraint	Amount	Slack	Unit Cost	Minimum Value	Maximum Value
Mn Met Energy	1400.cal/lb	0	.31	1369.	1409.
Mn Prd Energy	1000.cal/lb	28.19	0	**	1028.
Mn Fat	6.00%	0	96.48	5.478	8.351
Mx Fiber	5.00%	1.78	0	3.215	**
Mn Protein	22.00%	0	9.23	21.591	22.627
Mn Calcium	1.00%	.10	0	**	1.100
Mx Calcium	1.10%	0	64.43	1.000	1.316
Mn Phosphorus	.45%	0	310.81	.319	.600
Mx Phosphorus	.60%	.15	0	.450	**
Mn Arginine	1.20%	.15	0	**	1.348
Mn Glycine	.84%	.19	0	**	1.034
Mn Lysine	1.10%	.05	0	**	1.150
Mn Methionine	.50	0	581.75	.494	.509
Mn Meth, Cystine	.85%	0	1783.44	.840	.856
Mn Tryptophane	.22%	.02	0	**	.242
Mn Xanthophyll	6.3mg/lb	0	7.88	2.906	7.787
Mx Barley, Milo	15.00%	0	1.69	0	20.152
Mx Corn Dist Sol	5.00%	5.00	0	0	**
Mx Corn Gluten	5.00%	5.00	0	0	**
Mx Crab Meal	5.00%	3.37	0	1.630	**
Mx Fish Meal	7.50%	7.50	0	0	**
Mx Poultry Meal	7.50%	0	4.3	3.572	13.836
Mx Stab Fat	8.00%	5.23	0	2.771	**
Salt, Premixes	.60%	0	540.19	0	1.241

Figure 9. Specification costs, week two

(for example, fish meal, X_8; meatscraps, X_{10}; and poultry meal, X_{14}) must be equal to or less than 18 percent of the plant origin, high protein ingredient (soybean meal, X_{15}). This constraint may be stated as follows:

$$X_8 + X_{10} + X_{14} \leq .18\, X_{15} \qquad (1\text{-}12)$$

That is, the amount of fish meal (X_8) plus the amount of meatscraps (X_{10}) plus the amount of poultry meal (X_{14}) shall be equal to or less than 18 percent of the amount of soybean meal (X_{15}). For purposes of computer analysis, the right-hand-side term (.18 X_{15}) is transposed to the left-hand-side to give the following:

$$X_8 + X_{10} + X_{14} - .18\, X_{15} \leq 0 \qquad (1\text{-}13)$$

This type of constraint permits accurate control of the overall use of a group of ingredients in the formula without unduly restricting the use of individual ingredients. Maximum flexibility of ingredient substitution is also allowed in meeting the overall formulation specifications.

Dual Constraints

The use of a dual constraint is illustrated in the single formula model in connection with the methionine and cystine requirements. These are typical of the type of nutritional interrelationships that need to be controlled with a dual constraint. The problem arises from the fact that methionine can satisfy the cystine requirement, but cystine cannot satisfy the methionine requirement. Formulation models that include only a methionine requirement and a cystine requirement overlook this important nutritional interrelationship, thereby imposing an economic penalty on the formula. Recognition is not given to the dual role being performed by methionine. This problem is solved through the use of a dual constraint.

Thus, in addition to the methionine requirement constraint, a dual constraint for methionine and cystine is included as follows:

Methionine Requirement

$$.003\, X_1 + .0017\, X_2 + \ldots + .003\, X_{17} \geq .5 \qquad (1\text{-}14)$$

Methionine and Cystine Requirement

$$.0065\ X_1 + .0034\ X_2 + \ldots . + .0062\ X_{17} \geq .85 \quad (1\text{-}15)$$

The omission of the latter dual constraint can increase formula costs by as much as $1 per ton.

Feed Density Constraints

The use of feed density constraints is a relatively recent development in the feed manufacturing industry. Nutrient requirements are stated in terms of each calorie of metabolizable energy in the feed. This ensures a uniform density of all essential nutrients for each calorie of metabolizable energy. Formulas are chosen by the model to give a minimum cost for total nutrients. The following illustrates the type of constraints being used:

Conventional Energy Control

Weight:

$$X_1 + X_2 + \ldots . + X_{17} = 100 \quad (1\text{-}16a)$$

Total ingredient use must equal 100 percent of a ton.

Metabolizable Energy:

$$6.4\ X_1 + 12.8\ X_2 + \ldots . + 8.3\ X_{17} \geq 1400 \quad (1\text{-}16b)$$

Caloric content of the feed must equal at least 1400 calories per pound or 2,800,000 calories per ton of feed. In equation (1-16b), the coefficients on the metabolizable energy restraint have been scaled to give energy control in terms of calories per pound for each pound of the ton of feed. Should the analyst desire to state the restraint in terms of calories per ton, he would express it as follows:

$$12,800\ X_1 + 25,600\ X_2 + \ldots .$$
$$+16,600\ X_{17} \geq 2,800,000 \quad (1\text{-}16c)$$

Both equations (1-16b and 1-16c) provide identical restraint on the formulation model. The latter equation (1-16c) specifies the number of calories required in a ton of feed. The former equation (1-16b) specifies the number of calories required in each pound of the ton of feed. For discussion purposes, the per pound form of the equation is used.

All other nutrient requirements of the formula may be expressed as a percent or portion of the weight of the feed.

Variable Density Energy Control

Metabolizable Energy:

$$6.4\ X_1 + 12.8\ X_2 + \ldots . + 8.3\ X_{17}$$
$$- X_{18} = 0 \quad (1\text{-}16d)$$

$$X_{18} = 1400 \quad (1\text{-}16e)$$

The total energy level in the formula is given by the value of variable X_{18} (1-16d). This level is set at 1400 calories (1-16e). No specification is made as to the weight of the feed containing the 1400 calories. Depending on relative ingredient costs, it may not take a pound of feed to obtain that calorie level. The model is designed to get 1400 calories with the least-cost combination of ingredients, subject to the other formulation restraints. Thus, it is possible for the calorie/weight ratio to exceed the previous 1400 calories per pound relationship.

The important ratios in this case are those relating to nutrient requirements. A uniform amount of all essential nutrients must be provided for each calorie of metabolizable energy in the feed. This is accomplished in the following manner:

Arginine Control:

$$.009\ X_1 + .005\ X_2 + \ldots . + .0036\ X_{17}$$
$$\geq .00085\ X_{18} \quad (1\text{-}17)$$

Arginine content must be equal to or greater than a specified fraction of total energy (X_{18}).

Glycine Control:

$$.009\ X_1 + .0035\ X_2 + \ldots . + .007\ X_{17}$$
$$\geq .0006\ X_{18} \quad (1\text{-}18)$$

Glycine content must be equal to or greater than a specified fraction of total energy (X_{18}).

When the model gives formulas with over 1400 calories per pound (it takes less than a pound of feed to get 1400 calories), the cost of total nutrients is reduced. Thus, the cost of feeding performance is reduced for a given nutrient input.

It is important to note that the non-nutrient requirements still remain tied to the weight of the feed to ensure uniformity of such items as color, texture, and medicaments. However, the nutrient density is allowed to vary as necessary to minimize total nutrient cost.

Blocking Constraints

Blocking constraints can be used to exclude quickly from the formula ingredients that are already built into a model. Only one constraint is needed for each model. The role of the constraint is to prohibit the use of any ingredient that is not available or that management does not wish to include in the formula. For example, if alfalfa meal (X_1), crab meal (X_6), and milo (X_{11}) are to be excluded, the blocking constraint can be stated as follows:

$$X_1 + X_6 + X_{11} = 0 \tag{1-19}$$

Another blocking technique commonly used is that of placing artificially high costs on the ingredients to be excluded from the formula. Unfortunately, this also distorts the collateral information relative to the excluded ingredients. This problem does not occur with the blocking constraint approach.

COMPOSITE FORMULA MODEL

Greatly increased computational efficiency is achieved through the use of the composite formula model. Normally, a feed manufacturer develops a small number of composite models to generate formulation guides for his entire line of feeds. Each composite model is used to represent a group of feeds that have much in common in terms of formulation specifications and ingredients used. To illustrate the use of this type of model, formulas for a group of six cattle, dairy, and hog supplements are analyzed.

FORMULATION SPECIFICATIONS

For this series of feed supplements, formulation specifications are given in Figures 10 through 15. The formulas, including the high protein, urea formulas, are designed to provide a supplement to feed grains. Specifications on ingredient utilization rates are used to control, in addition to protein, calcium, and phosphorus requirements, the formulation of cattle and dairy supplements. The hog supplements have additional requirements in terms of minimum levels of selected amino acids.

Calculated Analysis

Item	Minimum	Maximum
Protein	55.	55.
Calcium	...	3.
Phosphate	1.2	...
Salt, Vit, Min	3.8	3.8

Ingredients

Item	Minimum	Maximum
Alfalfa	1.75	13.5
Bran	1.	2.
Cottonseed meal	1.	30.
Dicalcium phosphate	...	...
Gluten feed	1.	18.
Limestone	...	...
Linseed	1.	30.
Cane molasses	1.	5.
Soybean, 50%	...	...
Middlings	1.	13.
Urea	9.	9.
Sodium sulphate	1.2	1.2

Figure 10. Formulation specifications, 55% cattle supplement

Calculated Analysis

Item	Minimum	Maximum
Protein	35.	35.
Calcium	...	1.86
Phosphate	.74	...
Salt, Vit, Min	3.6	3.6

Ingredients

Item	Minimum	Maximum
Alfalfa	1.75	13.5
Bran	1.	2.
Cottonseed meal	1.	30.
Dicalcium phosphate	...	...
Gluten feed	1.	14.
Limestone	...	...
Linseed	1.	30.
Cane molasses	1.	5.
Soybean, 50%	...	...
Middlings	1.	13.

Figure 11. Formulation specifications, 35% cattle supplement

Calculated Analysis

Item	Minimum	Maximum
Protein	55.	55.
Calcium	...	3.
Phosphate	1.2	...
Salt, Vit, Min	3.8	3.8

Ingredients

Item	Minimum	Maximum
Alfalfa	1.75	13.5
Bran	1.5	3.
Cottonseed meal	1.	30.
Dicalcium phosphate	...	...
Gluten feed	1.	18.
Limestone	...	...
Linseed	1.	30.
Cane molasses	1.	5.
Soybean, 50%	...	...
Middlings	1.	13.
Urea	9.	9.
Sodium sulphate	1.2	1.2

Figure 12. Formulation specifications, 55% dairy supplement

Calculated Analysis

Item	Minimum	Maximum
Protein	35.	35.
Calcium	...	1.86
Phosphate	.74	...
Salt, Vit, Min	3.6	3.6

Ingredients

Item	Minimum	Maximum
Alfalfa	1.75	13.5
Bran	1.	3.
Cottonseed meal	1.	30.
Dicalcium phosphate	...	...
Gluten feed	1.	14.
Limestone	...	...
Linseed	1.	30.
Cane molasses	1.	5.
Soybean, 50%	...	...
Middlings	1.	13.

Figure 13. Formulation specifications, 35% dairy supplement

Calculated Analysis

Item	Minimum	Maximum
Protein	35.	35.0
Calcium	...	3.75
Phosphorus	1.5	...
Cystine	0.5	...
Lysine	2.2	...
Methionine	.6	...
Tryptophane	.4	...
Salt, Vit, Min	3.6	3.6

Ingredients

Item	Minimum	Maximum
Alfalfa	5.0	15.0
Cottonseed meal	1.0	5.0
Degossyp cottonsd meal	...	20.0
Dicalcium phosphate	...	...
Defluorinated rock phos	...	...
Fish meal	5.0	25.0
Fish solubles	2.0	5.0
Limestone	...	...
Linseed meal	...	7.5
Meatscraps	...	7.5
Cane molasses	1.0	5.0
Soybean meal, 50%	...	...
Middlings	1.0	15.0

Figure 14. Formulation specifications, 35% hog grower supplement

Calculated Analysis

Item	Minimum	Maximum
Protein	35.	35.0
Calcium	...	3.75
Phosphorus	1.5	...
Cystine	0.5	...
Lysine	2.2	...
Methionine	.6	...
Tryptophane	.4	...
Salt, Vit, Min	3.6	3.6

Ingredients

Item	Minimum	Maximum
Alfalfa	1.0	15.0
Cottonseed meal	...	5.0
Degossyp cottonsd meal	...	20.0
Dicalcium phosphate	...	...
Defluorinated rock phos	...	...
Fish meal	5.0	25.0
Fish solubles	2.0	5.0
Limestone	...	...
Linseed meal	...	7.5
Meatscraps	...	22.5
Cane molasses	1.0	5.0
Soybean meal, 50%	...	...
Middlings	1.0	15.0

Figure 15. Formulation specifications, 35% hog finisher supplement

The information needed for composite model building is identical to that required for the single formula model. Each of the ingredients used in the feed formula is expressed as a matrix column or activity. Formulation specifications are stipulated by the rows and the elements of the right-hand-side (RHS) column. A unique right-hand-side column is provided for each supplement formula considered. Specifically, the first RHS gives the requirements for Cattle Supplement 55, the second RHS gives the requirements for Cattle Supplement 35, and so on (Figure 16).

In all cases, production of 100 tons of feed is assumed, although this figure can be changed as needed. The cost row coefficients are expressed in terms of dollars per ton. The coefficients of the row equations are expressed, with the exception of energy and xanthophyll requirements, in terms of percent or portion of process volume of feed produced. For energy control, row coefficients are scaled to give control in terms of calories per pound for each pound of feed in the total volume

	Alfalfa Meal (X_1)	Wheat Bran (X_2)	Cottonseed Meal (X_3)	Cottonseed Meal Deg. (X_4)	Dicalcium Phosphate (X_5)	Rock Phosphate (X_6)	Fish Meal Menhaden (X_7)	Fish Solubles (X_8)	Corn Gluten Feed (X_9)	Limestone (X_{10})	Linseed Meal (X_{11})	Meatscraps (X_{12})	Wheat Middlings (X_{13})	Molasses (X_{14})	Soybean Meal (X_{15})	Urea (X_{16})	Salt, Vit, Min (X_{17})		Cattle Supp 55	Cattle Supp 35	Dairy Supp 55	Dairy Supp 35	Hog Grower	Hog Finisher
Cost Control	58.	34.	66.	70.	82.	74.	127.	75.	40.	10.	64.	104.	34.5	45.	75.	100.	78.							
Weight Control	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	=	100.	100.	100.	100.	100.	100.
Protein Control	.2	.15	.42	.42			.6	.31	.222		.351	.499	.158	.03	.51	2.6		=	55.	35.	55.	35.	35.	35.
Calcium Mx	.017	.001	.0015	.0015	.23	.339	.05	.0014	.004	.38	.0033	.11	.001	.006	.0025			≤	3.	1.86	3.	1.86	3.75	3.75
Phosphorus Mn	.0009	.012	.011	.011	.187	.146	.025	.0065	.008		.0081	.06	.008	.0008	.002			≥	1.2	.74	1.2	.74	1.5	1.5
Cystine Mn	.0017	.0019	.0085	.0085			.01	.003	.0051		.0065	.006	.0019		.008			≥	0.	0.	0.	0.	.5	.5
Lysine Mn	.009	.006	.0155	.0155			.051	.021	.008		.013	.035	.007		.032			≥	0.	0.	0.	0.	2.2	2.2
Methionine Mn	.003	.0019	.0058	.0058			.018	.0093	.0031		.0078	.007	.002		.0075			≥	0.	0.	0.	0.	.6	.6
Tryptophane Mn	.0034	.0021	.005	.005			.0061	.0036	.0022		.0052	.0031	.002		.006			≥	0.	0.	0.	0.	.4	.4
Alfalfa Mn	1.																	≥	1.75	1.75	1.75	1.75	5.	1.
Alfalfa Mx	1.																	≤	13.5	13.5	13.5	13.5	15.	15.
Wheat Bran Mn		1.																≥	1.	1.	1.5	1.	0.	0.
Wheat Bran Mx		1.																≤	2.	2.	3.	3.	0.	0.
Cottonseed Mn			1.															≥	1.	1.	1.	1.	1.	0.
Cottonseed Mx			1.															≤	30.	30.	30.	30.	5.	5.
Cottonseed Deg. Mx				1.														≤	0.	0.	0.	0.	20.	20.
Fish Meal Mn							1.											≥	0.	0.	0.	0.	5.	5.
Fish Meal Mx							1.											≤	0.	0.	0.	0.	25.	25.
Fish Solubles Mn								1.										≥	0.	0.	0.	0.	2.	2.
Fish Solubles Mx								1.										≤	0.	0.	0.	0.	5.	5.
Corn Gluten Mn									1.									≥	1.	1.	1.	1.	0.	0.
Corn Gluten Mx									1.									≤	18.	14.	18.	14.	0.	0.
Linseed Mn											1.							≥	1.	1.	1.	1.	0.	0.
Linseed Mx											1.							≤	30.	30.	30.	30.	7.5	7.5
Meatscraps Mx												1.						≤	0.	0.	0.	0.	7.5	22.5
Middlings Mn													1.					≥	1.	1.	1.	1.	1.	1.
Middlings Mx													1.					≤	13.	13.	13.	13.	15.	15.
Molasses Mn														1.				≥	1.	1.	1.	1.	1.	1.
Molasses Mx														1.				≤	5.	5.	5.	5.	5.	5.
Urea Control																1.		≤	9.	0.	9.	0.	0.	0.
Salt, Vit, Min																	1.	=	3.8	3.6	3.8	3.6	3.6	3.6

Figure 16. Matrix tableau, composite formula model

processed. For xanthophyll control, row coefficients are scaled to give control in terms of milligrams per pound for each pound of feed in the total volume processed.

PROBLEM ANALYSIS

In the actual problem analysis, a least-cost formula is found for each of the RHS or formula specifications. Thus, in the case of Cattle Supplement 55, the least-cost combination of ingredients must contain 55 percent protein, a maximum of three percent calcium, and a minimum of 1.2 percent phosphorus (Figure 16). Specifications also are given in terms of utilization of individual ingredients: for example, alfalfa meal has a minimum use rate of 1.75 percent and a maximum of 13.5 percent; bran has a minimum use rate of one percent and a maximum of two percent; etc.

After the least-cost formula has been computed for this first RHS, another least-cost formula is calculated to meet the needs of the second RHS. This process is continued until least-cost formulas have been calculated for each RHS or feed.

The major reduction in computational time arises from the fact that the selected group of supplements uses common ingredients and has similar specifications. The least-cost formula for the first RHS is similar to the least-cost formula for the second RHS; the least-cost formula for the second RHS is similar to the least cost formula for the third RHS; etc. Once the initial least-cost formula has been obtained, a comparatively small amount of additional computing time is required to determine the remaining formulas. Analysis of the management guides data for these supplements reveals their inherent similarities (Figures 17 through 22).

Given careful analysis of these least-cost formulas over a particular period, it is possible to sequence the order of their solution to facilitate minimization of computational time. In addition, modifications can be made in the basic LP code to ensure that the optimum sequence is chosen by the computer (see reference 5).

LEAST-COST CATTLE SUPPLEMENT 55
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	1.75	35.53	***
WHEAT BRAN	34.00	1.00	33.51	***
COTTONSEED MEAL	66.00	30.00	***	67.13
DICALCIUM PHOSPHATE	82.00	3.43	59.40	90.00
CORN GLUTEN FEED	40.00	18.00	***	41.04
LIMESTONE	10.00	2.64	8.03	10.76
LINSEED MEAL	64.00	1.00	57.32	***
WHEAT MIDDLINGS	34.50	1.00	32.98	***
MOLASSES	45.00	1.00	14.09	***
SOYBEAN MEAL	75.00	27.38	73.63	76.66
UREA	100.00	9.00	***	337.45
SALT VITAMIN MINERAL	78.00	3.80	***	***

FORMULA COST PER TON 65.36

Figure 17. Least-cost 55% cattle supplement

LEAST-COST CATTLE SUPPLEMENT 35
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	1.75	36.81	***
WHEAT BRAN	34.00	2.00	***	35.05
COTTONSEED MEAL	66.00	30.00	***	67.49
DICALCIUM PHOSPHATE	82.00	.66	45.50	92.39
CORN GLUTEN FEED	40.00	14.00	***	42.26
LIMESTONE	10.00	3.88	***	12.34
LINSEED MEAL	64.00	1.00	57.99	***
WHEAT MIDDLINGS	34.50	9.13	33.46	45.20
MOLASSES	45.00	1.00	16.17	***
SOYBEAN MEAL	75.00	32.98	73.02	78.35
SALT VITAMIN MINERAL	78.00	3.60	***	***

FORMULA COST PER TON 59.80

Figure 18. Least-cost 35% cattle supplement

LEAST-COST DAIRY SUPPLEMENT 55
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	1.75	35.53	***
WHEAT BRAN	34.00	1.50	33.51	***
COTTONSEED MEAL	66.00	30.00	***	67.13
DICALCIUM PHOSPHATE	82.00	3.40	59.40	90.00
CORN GLUTEN FEED	40.00	18.00	***	41.04
LIMESTONE	10.00	2.32	8.03	10.76
LINSEED MEAL	64.00	1.00	57.32	***
WHEAT MIDDLINGS	34.50	1.00	32.98	***
MOLASSES	45.00	1.00	14.09	***
SOYBEAN MEAL	75.00	27.23	73.63	76.66
UREA	100.00	9.00	***	337.45
SALT VITAMIN MINERAL	78.00	3.80	***	***

FORMULA COST PER TON 65.36

Figure 19. Least-cost 55% dairy supplement

LEAST-COST DAIRY SUPPLEMENT 35
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	1.75	36.81	***
WHEAT BRAN	34.00	3.00	***	35.05
COTTONSEED MEAL	66.00	30.00	***	67.49
DICALCIUM PHOSPHATE	82.00	.64	45.50	92.39
CORN GLUTEN FEED	40.00	14.00	***	42.26
LIMESTONE	10.00	3.89	***	12.34
LINSEED MEAL	64.00	1.00	57.99	***
WHEAT MIDDLINGS	34.50	8.12	33.46	45.20
MOLASSES	45.00	1.00	16.17	***
SOYBEAN MEAL	75.00	33.00	73.02	78.35
SALT VITAMIN MINERAL	78.00	3.60	***	***

FORMULA COST PER TON 59.79

Figure 20. Least-cost 35% dairy supplement

LEAST-COST HOG GROWER SUPPLEMENT 35
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	5.00	52.24	***
COTTONSEED MEAL	66.00	1.00	57.76	***
DICALCIUM PHOSPHATE	82.00	5.45	25.07	101.52
FISH MEAL	127.00	6.81	112.27	242.29
FISH SOLUBLES	75.00	5.00	***	81.36
LIMESTONE	10.00	5.01	***	46.10
LINSEED MEAL	64.00	1.96	46.87	74.04
WHEAT MIDDLINGS	34.50	15.00	***	53.50
MOLASSES	45.00	2.52	31.21	55.98
SOYBEAN MEAL	75.00	48.65	26.04	88.56
SALT VITAMIN MINERAL	78.00	3.60	***	***

FORMULA COST PER TON 67.78

Figure 21. Least-cost 35% hog grower supplement

LEAST-COST HOG FINISHER SUPPLEMENT 35
DECEMBER 10,196-

FEED INGREDIENTS IN THE FORMULA	PRICE PER TON	PERCENT USED IN FORMULA	PRICE RANGES TO KEEP FORMULA LEAST-COST (LOWER)	(UPPER)
ALFALFA MEAL	58.00	1.00	52.24	***
DICALCIUM PHOSPHATE	82.00	5.49	25.07	101.52
FISH MEAL	127.00	6.38	112.27	242.29
FISH SOLUBLES	75.00	5.00	***	81.36
LIMESTONE	10.00	5.17	***	46.10
LINSEED MEAL	64.00	3.71	46.87	74.04
WHEAT MIDDLINGS	34.50	15.00	***	53.50
MOLASSES	45.00	4.42	31.21	55.98
SOYBEAN MEAL	75.00	50.23	26.04	88.56
SALT VITAMIN MINERAL	78.00	3.60	***	***

FORMULA COST PER TON 67.47

Figure 22. Least-cost 35% hog finisher supplement

MULTIFORMULA MODEL

Whenever limitations arise in ingredient supplies, in plant production capacity, or in any other common component of the manufacturing operation, the use of a multiformula model becomes a necessity. In such situations it is necessary to allocate the limited resources among alternative products. Many of the important allocation and related operating problems facing management are left unanswered by the use of a single formula model. Arbitrary allocation of limited inputs inevitably leads to suboptimization with its attendant higher cost. Clearly, a need exists for the simultaneous analysis of allocation alternatives as provided by the multiformula model.

MATRIX SIZE CONSIDERATIONS

Excessive matrix size constitutes a potential problem in the use of the multiformula model. It is all too easy to develop a model for which computational costs become prohibitive. Fortunately, matrix reduction can usually be accomplished without appreciable loss of accuracy. Among the obvious areas of reduction are the following: trivial restrictions, arbitrary restrictions, and the inclusion of fixed-level ingredients. Reduction in other areas inevitably becomes possible as the analyst becomes better acquainted with the operations represented by the model. Moreover, the use of high speed computers and advanced LP programs enables the analyst to solve problems containing over 100 equations in less than two minutes. Current industry use indicates that with respect to computational cost per formula, multiformula models can be competitive with single or composite formula models. In light of the additional management guides they provide, multiformula models are unquestionably superior where ingredient or production limitations exist.

For purposes of initial discussion, assume that a multiformula model is built for a firm that uses six ingredients to produce three feeds (Figure 23). The ingredients are available in given quantities from two sources, namely: (1) regular order rail delivery, or (2) special order truck delivery. Of the three feeds manufactured, cattle supplement and hog grower ration have formulation specifications similar to those discussed previously. The third feed, goat ration, must use one of two standard formulas. Production requirements for all feeds also are specified.

Ingredients Available

Regular Delivery

Item	Price (dollars/ton)	Quantity (tons)
Alfalfa Meal	59.00	300
Corn Meal	54.00	500
Cottonseed Meal	66.00	425
Soybean Meal	79.00	500
Meatscraps	86.00	375
Middlings	35.00	175

Special Delivery

Item	Price (dollars/ton)	Quantity (tons)
Alfalfa Meal	61.00	150
Soybean Meal	82.00	200
Meatscraps	89.00	125

Formulation Specifications

Cattle Supplement

1. Ingredients used: alfalfa meal, cottonseed meal, soybean meal, middlings
2. Minimum protein of 20% of supplement
3. Maximum fiber of 10% of supplement

Hog Grower Ration

1. Ingredients used: alfalfa meal, corn meal, cottonseed meal, soybean meal, meatscraps
2. Minimum protein of 17% of ration
3. Maximum fiber of 9% of ration

Goat Ration

Formula One: corn meal, 65%; cottonseed meal, 10%; soybean meal, 25%

Formula Two: corn meal, 50%; cottonseed meal, 15%; soybean meal 35%

Production Requirements

Cattle Supplement	800 tons
Hog Grower Ration	950 tons
Goat Ration	75 tons

Figure 23. Input data, multiformula model

MODEL FORMULATION

The multiformula model developed for this simplified problem is given in Figure 24. Its major components are summarized in the following submatrices: ingredient supply, ingredient use, formulation control, and production requirements control.

Ingredient Supply

The ingredient supply submatrix has as activities or columns all ingredients that are available to the firm. This includes six regular purchase, rail-delivered ingredients and three special purchase, truck-delivered ingredients. Constraints on the permissible level of purchase are placed on each ingredient to reflect the amounts available to the firm.

$$\begin{array}{cccccccc}
\text{ALFML} & \text{CORN} & \text{COTTN} & \text{SOYML} & \text{MEAT} & \text{MIDDS} & & \text{RHS} \\
X_1 & & & & & & \le & 300 \\
 & X_2 & & & & & \le & 500 \\
 & & X_3 & & & & \le & 425 \\
 & & & X_4 & & & \le & 500 \\
 & & & & X_5 & & \le & 375 \\
 & & & & & X_6 & \le & 175
\end{array} \qquad (2\text{-}1)$$

Constraints of this type consist of a coefficient of one in the appropriate ingredient column and the maximum amount available in the right-hand-side column. Thus, the permissible levels of purchase are limited to a maximum of 300 tons for alfalfa meal (X_1), 500 tons for corn meal (X_2), 425 tons for cottonseed meal (X_3), etc.

The use of this simple technique ensures that ingredient purchase activities conform to general market availability and current inventory levels of ingredients. The technique also may be used to force a minimum use level of specified ingredients. For example, should the supply of cottonseed meal be abnormally large, it might be desirable to specify the use of at least 250 tons of the meal, in which case the constraint would be written as follows:

$$\begin{array}{ccc}
\text{COTTN} & & \text{RHS} \\
X_3 & \ge & 250
\end{array} \qquad (2\text{-}2)$$

In using minimum allocation constraints, caution must be exercised to ensure that the minimum amount specified (RHS value of 250) is, in fact, feasible. In the problem stated, the cattle and hog rations have maximum fiber contents of ten and nine percent, respectively, whereas cottonseed has a fiber content of 16 percent. It is obvious that the use of cottonseed meal in the ration must be balanced with the

		Alfalfa RP	Corn RP	Cottonseed RP	Soybean RP	Meatscraps RP	Middlings RP	Alfalfa SP	Soybean SP	Meatscraps SP	Alfalfa Use Cattle	Cottonseed Use Cattle	Soybean Use Cattle	Middlings Use Cattle	Cattle Supplement	Alfalfa Use Hog	Corn Use Hog	Cottonseed Use Hog	Soy Use Hog	Meat Use Hog	Hog Grower	Formula 1 Goat	Formula 2 Goat	
		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	
	Cost Control	59.	54.	66.	79.	86.	35.	61.	82.	89.														
INGREDIENT SUPPLY	Alfalfa RP Supply	1.																						≤ 300
	Corn RP Supply		1.																					≤ 500
	Cottonseed RP Supply			1.																				≤ 425
	Soybean RP Supply				1.																			≤ 500
	Meatscraps RP Supply					1.																		≤ 375
	Middlings RP Supply						1.																	≤ 175
	Alfalfa SP Supply							1.																≤ 150
	Soybean SP Supply								1.															≤ 200
	Meatscraps SP Supply									1.														≤ 125
INGREDIENT USE	Alfalfa Transfer	1.									-1.					-1.								= 0
	Corn Transfer		1.														-1.					-.65	-.50	= 0
	Cottonseed Transfer			1.								-1.						-1.				-.10	-.15	= 0
	Soybean Transfer				1.				1.				-1.						-1.			-.25	-.35	= 0
	Meatscraps Transfer					1.				1.										-1.				= 0
	Middlings Transfer						1.							-1.										= 0
FORMULATION CONTROL	Weight, Cattle Supply										1.	1.	1.	1.	-1.									= 0
	Protein, Cattle Supply										.2	.42	.50	.16	-.2									≥ 0
	Fiber, Cattle Supply										.18	.16	.07	.08	-.1									≤ 0
	Weight, Hog Grow															1	1	1	1	1	-1			= 0
	Protein, Hog Grow															.2	.08	.42	.50	.50	-.17			≥ 0
	Fiber, Hog Grow															.18	.08	.16	.07	.03	-.09			≤ 0
PRODUCTION REQUIREMENTS	Cattle Supply Production														1									= 800
	Hog Grow Production																				1			= 950
	Goat Ration Production																					1	1	= 75

Figure 24. Matrix tableau, multiformula model

use of low fiber ingredients. However, if the minimum use for cottonseed meal is set too high, it will not be possible to meet both the minimum cottonseed use requirement and the maximum fiber use requirement of the rations. Usually, infeasibilities of this type can be recognized by visual inspection of the problems.

Ingredient Use

The ingredient use submatrix, balances purchases of ingredients with their actual use in feed production. A coefficient of one is placed in each activity or column representing purchase of ingredients. Minus coefficients are placed in each column representing use of raw materials. The basic relationship may be expressed as follows:

Ingredient Purchase = Ingredient Use

This also may be stated as:

Ingredient Purchase - Ingredient Use = 0

The latter form is used in the matrix tableau. For example:

Alfalfa Meal Use

X_1	+ X_7	- X_{10}	- X_{15}	RHS = 0
Alfalfa Regular Purchase	Alfalfa Special Purchase	Alfalfa Used in Cattle Supplement	Alfalfa Used in Hog Supplement	

(2-3)

Similar controls are used for other ingredients. Note that ingredients used in the goat rations are controlled by standard formula specifications, and that they are transferred in specified amounts. This is explained in more detail in the following section.

Formulation Control

The formulation submatrix contains all specifications for feed formulation. In this model, ingredient specifications are expressed in terms of a percent or fraction of the finished product. Consider the following examples:

Cattle Supplement

Only three specifications are given, namely: weight control, minimum protein, and maximum fiber.

Weight control is given by the following equation:

$$X_{10} + X_{11} + X_{12} + X_{13} - X_{14} = 0 \qquad (2\text{-}4)$$

This specifies that alfalfa meal use (X_{10}) plus cottonseed meal use (X_{11}) plus soybean meal use (X_{12}) plus middlings use (X_{13}) is equal to cattle supplement production (X_{14}).

Protein control is given by the following equation:

$$.2X_{10} + .42X_{11} + .50X_{12} + .16X_{13} - .2X_{14} \geq 0 \qquad (2\text{-}5)$$

This specifies that amount of protein (20 percent) in alfalfa meal times amount used (X_{10}) plus .... plus amount of protein (16 percent) in middlings times amount used (X_{13}) shall be equal to or greater than 20 percent of the cattle supplement (X_{14}).

Fiber control is given by the following equation:

$$.18X_{10} + .16X_{11} + .07X_{12} + .08X_{13} - .1X_{14} \leq 0 \qquad (2\text{-}6)$$

This specifies that the amount of fiber (18 percent) in alfalfa meal times amount used (X_{10}) plus .... plus amount of fiber (eight percent) in middlings times amount used (X_{13}) shall be equal to or less than ten percent of the cattle supplement (X_{14}).

The same technique is used for hog grower formulation control. In general, all types of formulation specifications can be handled in a similar manner. The chief advantage of this particular model is the ease with which changes can be made for varying levels of total feed production. In addition, the basic format of this model is a required feature of the multiformula models that are used for analysis of marketing strategy problems.

Standard Formula Control

Standard formula specifications are sometimes used for low volume speciality items where the formulation alternatives are limited to several given choices. These can be handled by means of the ingredient use and ration activities (or columns) in the following goat ration formulas:

CORN Transfer	COTTN Transfer	SOYML Transfer	GOAT RATION Formula One	GOAT RATION Formula Two		RHS
$X_2 - X_{16}$			$-.65X_{21}$	$-.50X_{22}$	=	0
	$X_3 - X_{11} - X_{17}$		$-.10X_{21}$	$-.15X_{22}$	=	0
		$X_4 + X_8 - X_{12} - X_{18}$	$-.25X_{21}$	$-.35X_{22}$	=	0

(2-7)

The model specifies that corn meal use must be equal to 65 percent of the ration (formula one) or 50 percent of the ration (formula two). Cottonseed meal use must be equal to ten percent of the ration (formula one) or 15 percent of the ration (formula two). And soybean meal use must be equal to 25 percent of the ration (formula one) or 35 percent of the ration (formula two).

Either or both formulas may be used in the production of goat ration, depending upon the optimum allocation of the ingredients available for feed production. It is important to include these standard formula products in the model to ensure that ingredient allocation is truly optimum.

Production Requirements

The production requirements submatrix specifies the level of production for each feed. The requirements must be set here in the light of plant capacity as well as of market needs. In those situations where market sales potential exceeds plant capacity for any sales period, it is necessary to use the market allocation models developed in the following section.

In the problem considered, production requirements constraints are imposed by placing a coefficient of one in each column representing feed production and placing the amount to be produced in the RHS column as follows:

CATTS	HOGRT	GOAT ONE		GOAT TWO		RHS
X_{14}					=	800
	X_{20}				=	950
		X_{21}	+	X_{22}	=	75

(2-8)

Other Areas

Other areas of ingredient or production restrictions can be handled in the same way as that illustrated here. To date, models used by the industry have included allocation of in-plant carryover, demurrage-free inventory, demurrage inventory, and overall plant capacity.

INTRODUCTION

The models discussed have been oriented towards obtaining least-cost formulas for feed products. They have a proven record of cost reduction achievement in the feed manufacturing industry. Yet, in certain cases the least-cost formulas so derived are not the most profitable ones to use. The point is that in addition to the fact that they influence ingredient costs, the formulas used also influence labor costs, product line selection, and pricing policy — and all these factors determine contribution to profit and overhead. These other relationships are not considered in the traditional least-cost formulation model even though they may affect computed ingredient cost savings. A recent case study, using an LP model, indicates that it may be worth over $5,000 per week per plant to evaluate the neglected decision areas related to formula selection (see reference 1).

The LP model developed includes the major management decisions in the areas of procurement, production, and marketing. For purposes of discussion, it has been designated the Programmed Profit Analysis Model (PPAM), since it includes the major factors related to profit performance in feed manufacturing. As in the case of the formula models, PPAM provides for routine analysis of the feed manufacturing operations on a daily or weekly basis. Specific management reports that can be directly generated from the system include the following: estimated operation statements; marketing guides on pricing, promotion, and product mix; and production guides on raw material buying, labor utilization, and product formulation. Analysis also can be made of working capital management and the most profitable level of production.

The criterion for effectiveness of operation is maximization of short-run contribution to profit and overhead, where short-run profit maximization is not inconsistent with such long-run policies of the firm as marketing strategy, labor relations, and product quality.

ANATOMY OF DECISION VARIABLES

One important aspect of profitable management decision-making, hence of effective model building, is the separation of the controllable variables of the operation from the uncontrollable. Making the separation is admittedly difficult, because variables differ with the size and type of the company, the strength of the company's market position, the amount of customer loyalty to the company's products, and the length of the planning or decision period. Yet, despite the difficulties, the separation of the variables is a necessary first step.

For some firms, major controllability may emphasize formulation cost reduction, operating at a low-cost volume level, and meeting price competition. For other firms, controllability may emphasize such additional areas as product promotion, product mix, and labor utilization. The fact is that management has a wide range of decision variables to enable it to adjust profitably to the dynamic, yet largely uncontrollable, environment in which it operates.

In the PPAM tested, a group of eight controllable decision variables was identified (Figure 25). This categorization is an accurate representation of actual conditions faced by many firms. However, it is recognized that significant variations from this pattern do exist. In such cases, the basic PPAM can readily be adapted to include the variations. It is recognized also that decision variables that are uncontrollable over a weekly period may indeed be controllable over a longer period.

On examination it becomes apparent that the decision variables included in the system are closely interrelated (Figure 25). Decisions on least-cost formulas cannot be made without considering raw material availability. Raw material procurement must be related to product mix and product demand. Product mix is closely related to pricing and promotional policies are well as to labor and plant capacity. The use of working capital in the various production and sales activities must be analyzed and controlled in the light of competing product and sales alternatives. The level of output and the resultant contribution to profit and overhead are determined directly by decisions made in each of those interrelated areas.

A schematic representation of the PPAM is given in Figure 26. The mathematical structure of this advanced model is very similar to the multiformula model. The essential difference is the increased size and the expanded number of types of activities represented by the model. Large computers and model sizes of 300 to 400 equations have been used in current applications.

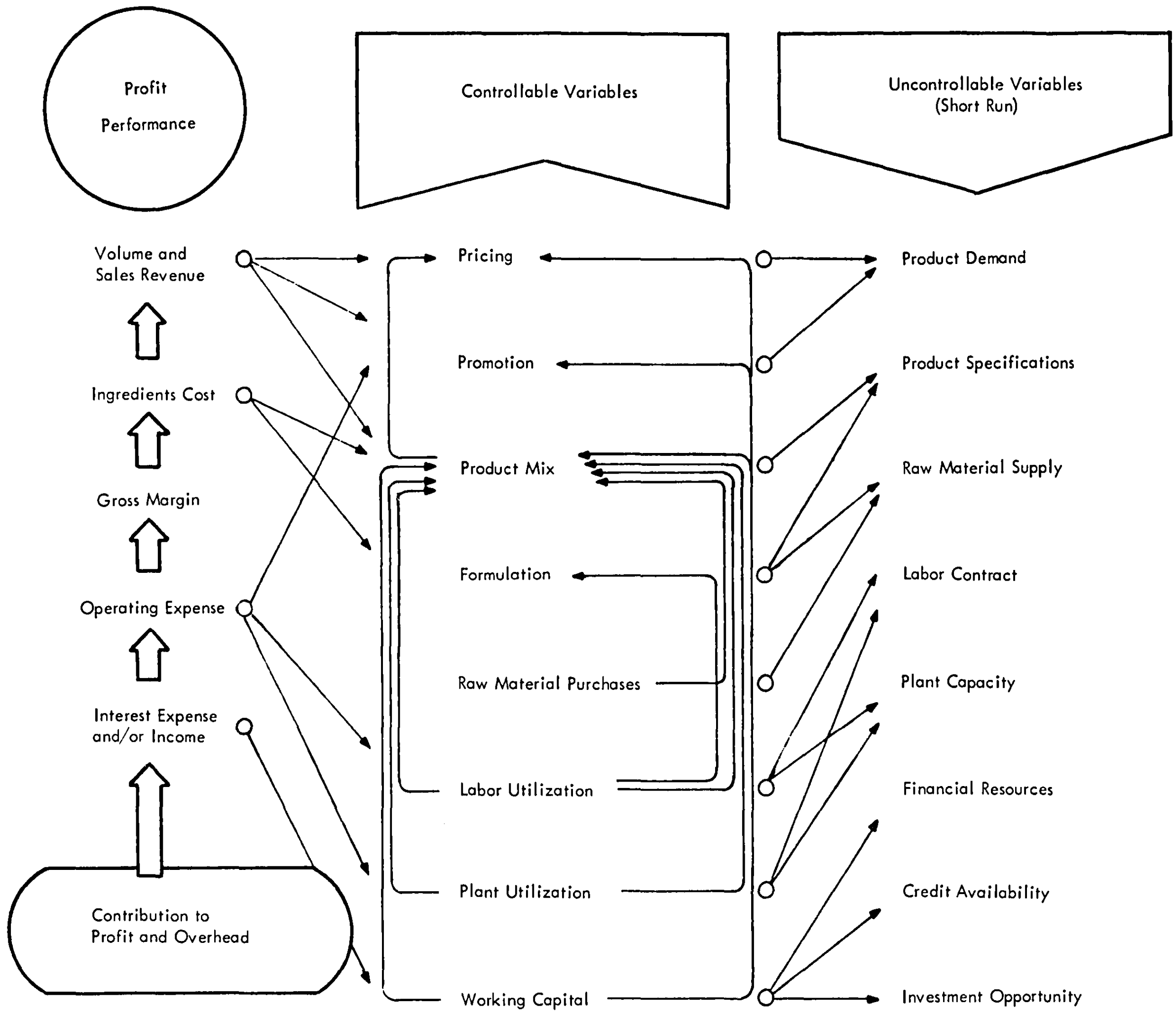

Figure 25. Major decision areas, programmed profit analysis model

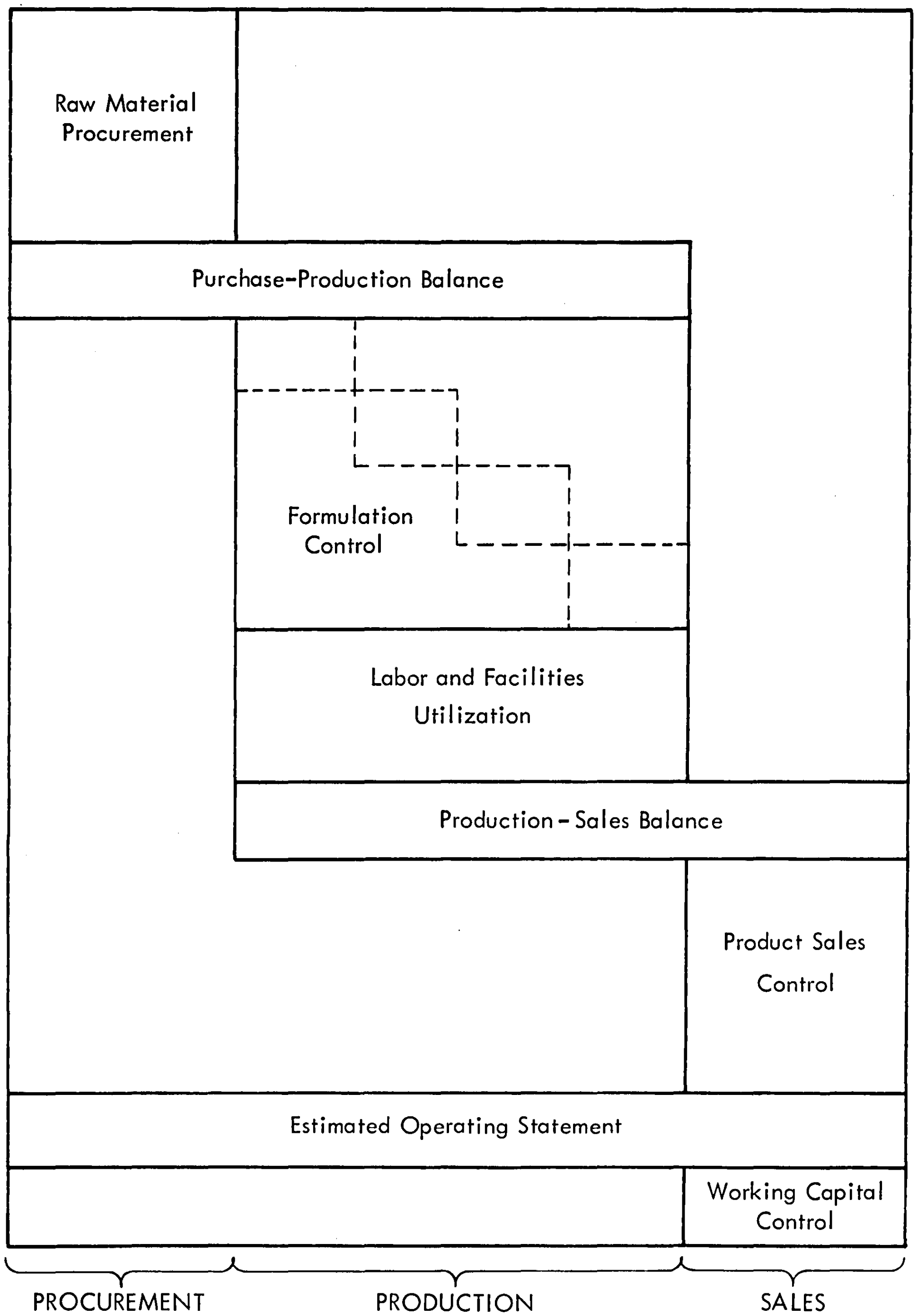

Figure 26. Matrix tableau, programmed profit analysis model

The single-formula example output (Figures 4, 5, 6) was generated by an output report writer for the IBM 1620.

The sample problem, if solved on the 1620/1311 LP system, would generate output reports that would differ slightly but which would still contain a great portion of the output report data given in Figures 4, 5, and 6.

The form of these reports is shown below. The mnemonics or symbols used are those for activities and matrix rows from Figure 2 (with N or X following the mnemonic for minimum or maximum constraint respectively).

OPTIMAL SOLUTION

Basis Variables Report

The symbol for each ingredient in the optimal feed is listed in this report (Figure 27) in the NAME column. The optimal level for each variable is listed in the corresponding position in the ACTIVITY LEVEL column.

BASIS VARBLS

NAME	ACTIVITY LEVEL	(1620)
ALFML	61.00	
CORN	54.00	
GLTML	86.00	
LIMST	9.00	
MEATS	88.00	
MILO	46.00	
METHN	2100.00	
OATS	52.00	
PLTML	103.00	
SOYML	84.00	
STFAT	138.00	
SALT	575.00	

Figure 27. Basis variables - Optimal output

Other Optimal Output

The slack variable for each constraint row and the cost of the formula are listed in this report (Figure 28).

The names of the slack variables formed to make equalities from inequalities are listed under the column heading NAME. The linear programming system has given these variables the names of the inequalities with which they are associated.

The actual slack activity levels computed are listed under the column heading ACTIVITY LEVEL. These indicate by how much the solution differs from the maximum or minimum levels given in the constraints.

The marginal costs of introducing nonoptimal slacks into the solution are listed under the column heading SIMPLEX MULT. The marginal cost is the cost of introducing one unit of the slack into the solution, while reducing one unit of another variable in the solution so that the constraint which corresponds to the slack does not meet its bound (maximum or minimum).

SLACKS

NAME	ACTIVITY LEVEL	SIMPLEX MULT.
COST	7187.	
METENN		.49
PRDENN	29.	
FATN		87.29
STFATX	5.	
FIBERX	1.80	
PROTNN		22.42
CALCMN	.10	
CALCMX		63.32
PHOSPN		244.93
PHOSPX	.15	
ARGINN	.12	
GLYCNN	.34	
LYSINN		58.48
METHNN	.033	
MECYSN		2066.62
TRYPTN	.003	
XANTHN		3.90
BARLYX		6.08
DISTSX	5.00	
GLTMLX	2.37	
CRBMLX	5.00	
FSHMLX	7.50	
PLTMLX		3.25
SALTN		541.62

Figure 28. Slacks - Optimal output

POSTOPTIMAL OUTPUT

DO.D/J Report

This report (Figure 29) contains the names and prices of ingredients not used in the formula (in the NAME and CURRENT COST columns respectively). The reduced costs, or the amounts by which the nonoptimal ingredient costs would have to be lowered before they could tie for entry into the solutions, are given in the REDUCED COST column. The cost levels for the various ingredients are given in the BASIS VALUE column.

DO.D/J

NAME	CURRENT COST	REDUCED COST	BASIS VALUE
BARLY	63.	18.05	44.95
DISTS	75.	1.66	73.34
CRBML	54.	.98	53.02
DPHOS	72.	11.00	61.00
FSHML	125.	.15	124.85
WHEY	125.	71.33	53.67

Figure 29. DO.D/J - Postoptimal output

Cost Range Report

For each activity (column) used in the optimal solution, this report (Figure 30) indicates the following data: current cost, highest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that highest cost, lowest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that lowest cost. When one of the entering variables is a bounded slack, the report indicates that at the critical price the activity level of the variable associated with the slack changes so that it no longer equals the restraining bound.

COST.R

NAME	CURRENT COST	HIGHEST COST	HI VAR	LOW VAR	LOWEST COST
ALFML	61.	61.53	FSHML	LYSINN	58.68
CORN	54.	54.16	FSHML	LYSINN	51.96
GLTML	86.	87.09	LYSINN	FSHML	85.78
LIMST	9.	12.13	CRBML	FSHML	2.89
MEATS	88.	88.22	FSHML	PLTMLX	79.80
MILO	46.	52.08	BARLYX	---	---
METHN	2100.	2110.78	FSHML	CRBML	1843.75
OATS	52.	53.51	LYSINN	FSHML	51.88
PLTML	103.	106.25	PLTMLX	---	---
SOYML	84.	84.14	FSHML	LYSINN	82.55
STFAT	138.	141.05	FSHML	PHOSPN	106.00
SALT	575.00	575.00	---	---	---

Figure 30. Cost ranges - Postoptimal output

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