COMPUTER TECHNOLOGY

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1620 USERS GROUP PROGRAM REVIEW AND EVALUATION

(fill out in typewriter or pencil, do not use ink)

Program No._____   Date_____

Program Name:_____

1. Does the abstract adequately describe what the program is and what   Yes___ No___
    it does?
    Comment_____

2. Does the program do what the abstract says?   Yes___ No___
    Comment_____

3. Is the Description clear, understandable, and adequate?   Yes___ No___
    Comment_____

4. Are the Operating Instructions understandable and in sufficient detail?   Yes___ No___
    Comment_____
    Are the Sense Switch options adequately described (if applicable)?   Yes___ No___
    Are the mnemonic labels identified or sufficiently understandable?   Yes___ No___
    Comment_____

5. Does the source program compile satisfactorily (if applicable)?   Yes___ No___
    Comment_____

6. Does the object program run satisfactorily?   Yes___ No___
    Comment_____

7. Number of test cases run_____. Are any restrictions as to data,
    size, range, etc. covered adequately in description?   Yes___ No___
    Comment_____

8. Does the Program Meet the minimal standards of the 1620 Users
    Group?   Yes___ No___
    Comment_____

9. Were all necessary parts of the program received?   Yes___ No___
    Comment_____

10. Please list on the back any suggestions to improve the usefulness of the program.
    These will be passed onto the author for his consideration.

Please return to: Your Name _____

    Mr. Richard L. Pratt   Company   _____
    Data Corporation
    7500 Old Xenia Pike   Address   _____
    Dayton, Ohio   45432
                               User Group Code_____

THIS REVIEW FORM IS PART OF THE 1620 USER GROUP ORGANIZATION'S PROGRAM
REVIEW AND EVALUATION PROCEDURE. NONMEMBERS ARE CORDIALLY INVITED
TO PARTICIPATE IN THIS EVALUATION.

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11/09/64

Portfolio Selection-Diagonal Model

1. Program Deck

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Program Abstract

Title (If subroutine state in Title) PORTFOLIO SELECTIONS - DIAGONAL MODEL
   FOR IBM-1620 (40K)
                                        Subject Classification   10.3

Author: Organization: PAO L. CHENG and HUGH R. GARDNER, School of Business
   Administration, University of Mass., Amherst, Mass.

Direct Inquires to:
Name Prof. Pao Lun Cheng   Address School of
   Business Administration, Univ. of Mass., Amherst   Phone   545-2568
                                         Mass.

Purpose/Description: The use of a simplified model to trace a critical line
   of efficient portfolio diversification.

Mathematical Method: Diagonal Model

Restrictions, Range: 80 securities may be processed

Storage Requirements: _____

Equipment Specifications:
        Memory 20K ___   40K x   60K ___   K ___ Automatic Divide: Yes x   No __
        Indirect Addressing: Yes x   No ___   Other Special Features Required __

Additional Remarks (Include at author's discretion: Language; Fixed/Float; Relocat-
ability) (Optional: Running time; Approximate number of times run successfully; Pro-
gramming Hours) Fortran II

4) Detailed Description of Program

   (a) Results and Methods

      Since this program is a translation of William F. Sharpe's 7090 program, the results and methods can be found in his article : "A Simplified Model for Portfolio Analysis", Management Science, Vol. 9, No. 2, Jan. 1963, pp. 277-293. The program can compute up to 80 securities for IBM-1620-40K. For each increase of 20K, one may expect to admit about 200 additional securities, provided appropriate changes in Dimension statements are made. The inputs are Y_1, Y_2, A_i, B_i, Q_i, $(i = 1, 2, \ldots n, n + 1)$,

where:

   Y_1 is the lending rate of interest,

   Y_2 is the borrowing rate of interest,

   A_i is the intercept of the rate of return of the ith security correlated with an index rate of return, or simply an index, (for $i = 1, 2, \ldots$ n).

   B_i is the slope of the rate of return of the ith security correlated with an index rate of return, (for $i = 1, 2, \ldots$ n).

   Q_i is the variance of the random variable associated with the correlation between the ith security and the index rate of return, (for $i = 1, 2, \ldots, n$). The mean of Q_i is zero. Q_i should be greater than zero as input.

   n   is the number of securities.

   A_{n+1} is the expected rate of return of the index.

   B_{n+1} is always zero.

   Q_{n+1} is the variance of the rate of return of the index. It must be greater than zero as input.

     The outputs are a set of corner portfolios, beginning with the corner portfolio that has the largest expected rate of return. The first corner portfolio would only consist of one security if among the n securities, there is one and only one security that has the highest expected rate of return.

The number of corner portfolios depends upon the amount of securities being processed and the lending rate specified at the outset. Given n, the higher the lending rate, the less the number of corner portfolios. In the absence of a specified lending rate, or $Y_1 = o$, the program produces the entire set of corner portfolios.

The program does not take account of the borrowing alternative in the manner used for the lending alternative. The program, however, does allow the user to specify the rate of interest at which funds can be borrowed. If this alternative is chosen, none of the corner portfolios which will be inefficient when borrowing is considered will be printed.

A corner portfolio is specified by a set of fractions, with the sum of the fractions equal to one. A particular fraction X_i is related to a particular i^{th} security, indicating that X_i of total funds be invested in the ith security.

   (b) Method of Computation

$$R_i = A_i + B_i I + C_i \qquad (i = 1,2,3,\ldots,n)$$

where A_i and B_i are as explained in (a). C_i is a random variable with an expected value of zero and variance Q_i. I is the level of some index.

$$I = A_{n+1} + C_{n+1}$$

where A_{n+1} is as explained in (a) and C_{n+1} is a random variable with an expected value of zero and a variance of Q_{n+1}. It is assumed that the covariance between C_i and C_j is aero for all values of i and j ($i = j$).

$$E_i = A_i + B_i (A_{n+1})$$
$$V_i = (B_i)^2 (Q_{n+1}) + Q_i$$
$$C_{ij} = (B_i) (B_j) (Q_{n+1})$$

where C_{ij} is the covariance between R_i and R_j.

For the expected return (E) and variance (V) of a portfolio, we have:

$$E = \sum_{i=j}^{n} X_i R_i$$

$$= \sum_{i=1}^{n} X_i (A_i + B_i I + C_i)$$

$$= \sum_{i=1}^{n} X_i (A_i + C_i) + \left[\sum_{i=1}^{n} X_i B_i \right] I$$

$$= \sum_{i=1}^{n} X_i (A_i + C_i) + X_{n+1} (A_{n+1} + C_{n+1})$$

$$= \sum_{i=1}^{n+1} X_i (A_i + C_i)$$

where we define $X_{n+1} = \sum_{i=1}^{n} X_i B_i$ and recognize $I = A_{n+1} + C_{n+1}$.

$$V = \sum_{i=1}^{n+1} X_i^2 Q_i$$

we maximize the objective function

$$\phi = \lambda E - V$$

where $E = \sum_{i=1}^{n+1} X_i A_i$

$$V = \sum_{i=1}^{n+1} X_i^2 Q_i$$

subject to $X_i \gtreqless 0$ for all i from 1 to n + 1.

$$\sum_{i=1}^{n} X_i = 1$$

$$\sum_{i=1}^{n} X_i B_i = X_{n+1}$$

The major steps of computation may be described by quoting a part of Sharpe's article (op.cit., p. 280):

"1. The corner portfolio with $\lambda = \infty$ is determined. It is composed entirely of the one security with the highest expected return.*
2. Relationships between (a) the amounts of the various securities contained in efficient portfolios and (b) the value of λ are computed. It is possible to derive such relationships for any section of the E, V curve between adjacent corner portfolios. The relationships which apply to one section of the curve will not, however, apply to any other section.

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*In the event that two or more of the securities have the same (highest) expected return, the first efficient portfolio is the combination of such securities with the lowest variance.

3. Using the relationships computed in (2), each security is examined to determine the value of λ at which a change in the securities included in the portfolio would come about:
 a. securities presently in the portfolio are examined to determine the value of λ at which they would drop out, and
 b. securities not presently in the portfolio are examined to determine the value of λ at which they would enter the portfolio.
4. The next largest value of λ at which a security either enters or drops out of the portfolio is determined. This indicates the location of the next corner portfolio.
5. The composition of the new corner portfolio is computed, using the relationships derived in (2). However, since these relationships held only for the section of the curve between this corner portfolio and the preceding one, the solution process can only continue if new relationships are derived. The method thus returns to step (2) unless $\lambda = 0$, in which case the analysis is complete."

It should be pointed out that in step 5, it is assumed that the lending rate is not present. If there is a specified lending rate, the computation of corner portfolio steps when λ is less than the slope of the line tangential to the "critical line" but passing through the point on E equal to the lending rate.

(c) See attached detailed flow chart.

Flowchart (hand-drawn, page rotated).

Top diagram (sheet 7):

(p,b)

(p,a) → Read 104: IM, AZ(I), BZ(I), QZ(I) → IF QZ(I) (+ / − / o)

- (−) → 220 ERR = 1.0
- (+ / o) → 224 Print 104: I, AZ(I), BZ(I), QZ(I) → IF ERR (+ / − / o)

(p,d)

IF ERR:
- (+) → 223 Print 115 → Go To 685
- (−) → 225 Print 113 → NSEC = M−1 → N1 = M → N2 = M+1 → N3 = M+2 → BZ(N1) = −1.0 → Do 250 I=1, N1

Do 250 I=1, N1:
- AB(I) = AZ(I)/QZ(I)
- BB(I) = BZ(I)/QZ(I)
- 250 DB(I) = 1.0/QZ(I)
- DB(N1) = 0.0

(M,a)

A

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Bottom diagram (sheet A):

START → Read 105: M → Read 100: Y1, Y2 → IF Y1 (+ / − / o)

- (−) → 202 Y1 = 1.0
- (+) → 201 IF Y2 (+ / − / o)
  - (−) → 203 Y2 = 1000.0
  - (+ / o) → 204 NUM = 1 → NHA = 0 → NSA = 0 → ERR = 0.0 → Print 101: Y1 → Print 102: Y2 → Print 103 → Do 221 I=1, M

(Y1, b)

(M, M)

(M, M)

A

SB = AZ(KME) * QB(KME) + AZ(N1) * BB(N1)

SD = AZ(KME) * DB(KME)

T(KME) = 1.0

T(N1) = BZ(KME)

Z1 = ZP/(ZP*ZL − ZN**2)

Z2 = −ZN/(ZP*ZL − ZN**2)

Z3 = 1.0/ZN

Z4 = −(Z1*ZL)/ZN

Z5 = −(Z2*ZL)/ZN

Z6 = Z3+Z4

T(N3) = −Z6

T(N4) = −Z1

U(N3) = Z1*SD + Z5*SB

U(N4) = Z1*SD + Z2*SB

ZMX = 0.0

KMX = KME−1

KP = KME+1

YME = 0.0

Do C 253 I=1,NSEG

E = AZ(I) + BZ(I) * HZ(I,KME)

IF E − YME

252 YME = E

KME = I

253 CONTINUE

Do 270 I=1,NSEG

270 ZI(I) = 0.0

ZI(KM1) = 1.0

ZN = 1.0/(TN(KME))

ZL = BQ(KME) − TZ

ZN = DB(KME) * BB(N1) + ZN

ZP = (BZ(KME)*BB(N1))/(QZ(KME)) + 1.0/QZ(N1)

Left column (page 10):

(pg. 9)

Do 276
I = 1, KMX

$ZRM = (T(N2) + BZ(I) * T(N3)) / (AZ(I) - U(N2) - BZ(I) * U(N3)$

IF ZRM − ZMX
−   +   0

275   ZMX = ZRM

KG = I

276 Continue

Do 278
I = KP, NSES

$ZRM = (T(N2) + BZ(I) * T(N3)) / (AZ(I) - U(N2) - BZ(I) * U(N3))$

IF ZRM − ZMX
−   +   0

277   ZMX = ZRM

KG = I

278 Continue

ZME = ZMX

(pg. 11)

Right column (page 11):

(pg. 10)

PRINT 111
NUM

E = YME

$VAR = QZ(KME) + (BZ(KME)**2) * QZ(N1)$

SDEV = VAR ** .5

Z2M = 2.0 * ZME

$ZIR = E - (2.0 * VAR) / Z2M$

PRINT 107
E, SDEV, ZIR

HUND = 100.0

PRINT 112
HUND, KME

PRINT 113

Go To 300

299
IF ZI(KG)
−   +   0

300
- $ZL = ZL + DB(KB)$
- $ZN = ZN + BB(KB)$
- $ZP = ZP + (BZ(KB)**2)/QZ(KB)$
- $SD = SD + AZ(KB)*DB(KB)$
- $SB = SB + AZ(KB)*BB(KB)$
- $ZI(KB) = 1.0$
- Go To 500

400
- $ZL = ZL - DB(KB)$
- $ZN = ZN - BB(KB)$
- $ZP = ZP - (BZ(KB)**2)/QZ(KB)$
- $SD = SD - AZ(KB)*DB(KB)$
- $SB = SB - AZ(KB)*BB(KB)$
- $ZI(KB) = 0.0$
- Go To 500

500
- $Z1 = \dfrac{ZP}{ZP+ZL-ZN**2}$
- $Z2 = \dfrac{-ZN}{ZP+ZL-ZN**2}$
- $Z3 = LD/ZN$
- $Z4 = -(Z1*ZL)/ZN$
- $Z5 = -(Z2*ZL)/ZN$
- $Z6 = Z3+Z4$
- $T(N3) = -Z6$
- $T(N2) = -Z1$
- $U(N3) = ZL*SD+Z5*SB$
- $U(N2) = Z1*SD+Z2*SB$

(Pg. 13)

Do 524 I=1, N1

IF ZI(KB) − / +

520
- $T(I) = 0.0$
- $U(I) = 0.0$

522
- $T(I) = Z1*DB(I)+ZL*BB(I)$
- $R = AB(I)-SD*(Z1*DB(I)+ZL*BB(I))$
- $S = SB*(Z2*DB(I)+Z5*BB(I))$
- $U(I) = R-S$

524 Continue

$ZMX = 0.0$

$KBX = KB$

Do 570 I=1, NSEC

IF ZI(I) − / +

560
$ZRM = \dfrac{T(N2)+BZ(I)*T(N3)}{AZ(I)-U(N2)-BZ(I)*U(N3)}$

IF ZRM-ZME

561 IF ZRM-ZMX

562 IF I-KBX

563 $ZMX = ZRM$

$KB = I$

550
$ZRM = -T(I)/U(I)$

IF ZRM-ZME +

552 IF ZRM-ZMX +

553 IF I-KBX

554 $ZMX = ZRM$

$KB = I$

570 Continue

(Pg. 14)

NMA=1
EX=0.0
SX=0.0
VX=0.0
ZLX=0.0

(P5,14)

602
$CX=(ZLAM-ZME)/(ELT-E)$
$BX=2.0*(ZME-CX*E)$
$AX=VAR-BX*E-CX*(E**2)$
$ELNT=\dfrac{-1.0+AX+BX*Y1}{2.0*CX*Y1+BX}$
$ZYL=BX*.5+CX*ELNT$

Do 608 I=1,N1

$X(I)=T(I)+U(I)*ZYL$
$EX=EX+X(I)*AZ(I)$
608 $VX=VX+QZ(I)*X(I)**2$

$SX=VX**.5$
$ZLX=2.0*ZYL$

IF NSA

40 PRINT 108 NUM,YA

612 PRINT 109
PRINT 107 EX,SX,YL
PRINT 110 ZLX
GO TO 640
(P9.18) 640

(P9.14)

580 ZLAM=ZME
IF ZMX
580 ZME=ZMX
ELT=E
E=0.0
VAR=0.0

Do 600 I=1,N1

$X(I)=T(I)+U(I)*ZME$
$VAR=VAR+QZ(I)*(X(I)**2)$
600 $E=E+X(I)*AZ(I)$

$SDEV=VAR**.5$
$Z2M=2.0*ZME$
$ZIR=E-(1.0+VAR)/ZAM$
IF ZIR-YL

GO TO 585 E=>

(P9.10) 14

(P9.15) 602

Flowchart (left side)

A

614 (NUM = NUM + 1)

IF ZIR-Y2 (- / +)

630 IF NSA

618 (NSA = 1)

(Go To 299)

632 PRINT 111 NUM

632 NSA = 0

N1UM = NUM-1

PRINT 107 E, SDEV, ZIA

PRINT 108 N1UM, Y2

PRINT 110 Z2M

640 Do 644 I=1, NSEC

IF X(I)

643 X(I) = 100. * X(I)

644 PRINT 112 X(I), I

CONTINUE

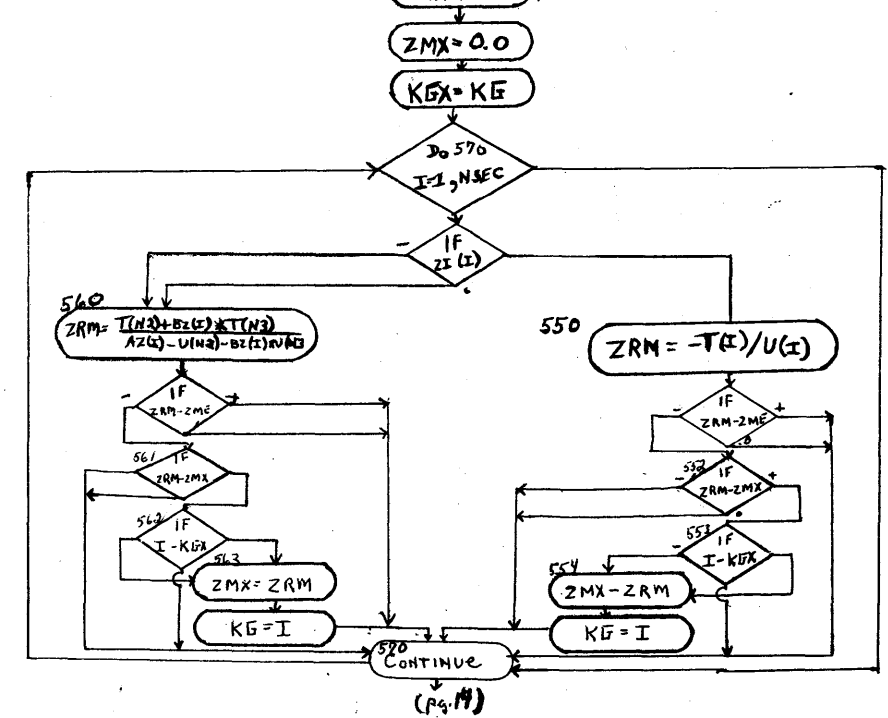PRINT 113

PRINT 113

685 IF NHA (- / +)

(Go To 299)

(pg. 11)

CONTINUE

STOP

END

INPUT/OUTPUT

1) Parameter Card (Input)

Cols.	Information
1 - 5	Number of securities (including the index) to be processed.

2) Parameter Card (Input)

Cols.	Information
1 - 10	The lending rate of interest.
10- 20	The borrowing rate of interest.

3) Parameter Cards (Input)

Cols.	Information
2 - 6	Dummy Variable
31- 40	Intercept of the rate of return of the ith security, correlated with an index rate of return, the last input is the expected rate of return of the index.
41- 50	Slope of the rate of return correlated with an index rate of return, but the last input is always zero, since it is an input related to the index.
51- 60	The variance of the random variable associated with the correlation between the ith security and the index rate of return, but the last input represents the variance of the index rate of return.

4) Labeled typewriter output